UC Merced: MATH 21 — Exam #2 — 2 November 2007

On the front of your bluebook print (1) your name, (2) your student ID number, (3) your discussion section number and instructor's name (Sprague or Lei), (4) a grading table, and (5) your seat number. Show all work in your bluebook and BOX IN YOUR FINAL ANSWERS where appropriate.
A correct answer with no supporting work may receive no credit while an incorrect answer with some correct work may receive partial credit. Textbooks, class notes, calculators and crib sheets are not permitted. There are a total of four problems on both sides of this paper and a total of 50 points. Please start each of the FOUR problems on a new page.
You have 50 minutes to complete the exam.

1. (10 points) Answer the following Always True (T) or False (F). Only your final answers will be graded on these problems.
   (a) If $f'(x) = g'(x)$ for all real numbers x, then $f(x) = g(x)$.
   (b) The velocity of a car is 58 mph at 4:00 PM, and is 66 mph at 4:15. At some time between 4:00 and 4:15, the acceleration of the car is 32 miles/hour^2.
   (c) If $f''(p) = 0$, then the graph $y = f(x)$ must have an inflection point at $x = p$.
   (d) The most general antiderivative of $\cos(x) - x$ is $\sin(x) - x^2/2$.
   (e) If $f(x)$ is integrable on $[a, b]$, then it is differentiable on (a, b).

2. (20 points) A function $f(x)$ is defined on $[-2, 3]$. $f(1) = 3$. The graph of the **derivative**, $f'(x)$, is given below.

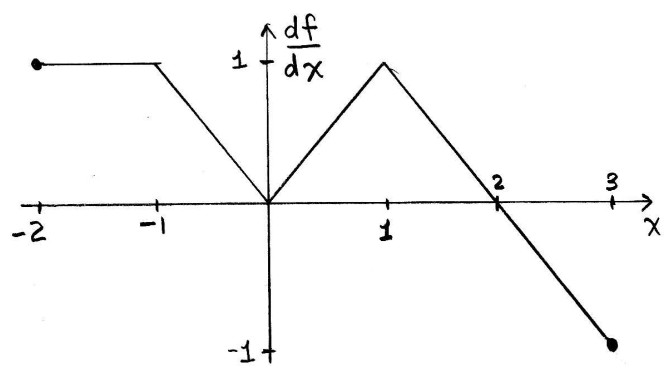

Figure 1: Problem 2

   (a) Find the linear approximation (also called the tangent-line approximation) of $f(x)$ at $x = 1$ and use it to estimate $f(0.98)$.
   (b) Find all critical points of $f(x)$ and determine whether each one is a local max, or a local min, or neither.
   (c) On what interval(s) is $f(x)$ concave up? concave down?
   (d) Sketch a graph of $f(x)$ reflecting your answers to part (b) and (c).

THERE'S MORE ON THE BACK!

3. (10 points) A baseball diamond is a square with side 90 ft. A batter hits the ball and runs toward first base with a speed of 24 ft/s as shown below. At what rate is his distance from the third base increasing when he is 45 ft from home plate? (To help your calculation, you are given that his distance to third base is $45\sqrt{5}$ft at that moment; you don't need to show this.)

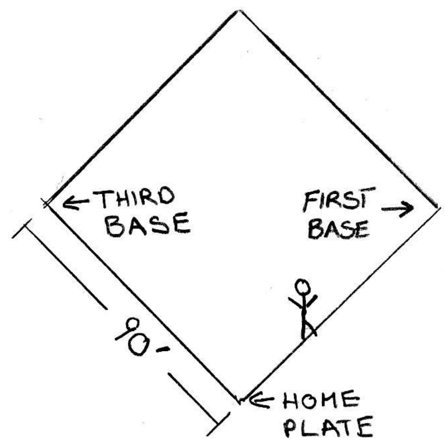

Figure 2: Baseball diamond for Prob. 3.

4. (10 points total) Answer the two following definite-integral problems:

   (a) What is the value of $\int_0^4 (x+1)\ dx$? Justify your answer.

   (b) Use a Riemann sum with 2 subintevals to approximate $\int_0^4 (x+1) dx$ with the Midpoint Rule.