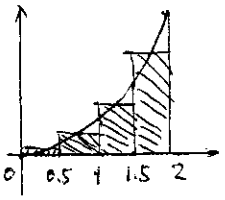
MATH21 — Solutions to Midterm 3 Fall 2008

1 (a) 

$\Delta x = \frac{2-0}{4} = \frac{2}{4} = \frac{1}{2} = 0.5$

midpoints: $\frac{0.5}{2} = 0.25$, $0.5 + 0.25 = 0.75$

$1 + 0.25 = 1.25$, $1.5 + 0.25 = 1.75$

$$\int_0^2 x^2\,dx \approx \boxed{0.5\left[(0.25)^2 + (0.75)^2 + (1.25)^2 + (1.75)^2\right]} = \frac{21}{8}$$

(b) $\int_0^2 x^2\,dx = \frac{1}{3}x^3\Big|_0^2 = \frac{1}{3}(2^3 - 0^3) = \boxed{\frac{8}{3}}$

3. $$\lim_{x\to 0}\frac{E(x)-\sin(x)}{x^2} = \lim_{x\to 0}\frac{\int_0^x e^{-t^2/2}\,dt - \sin(x)}{x^2} = \frac{\int_0^0 e^{-t^2/2}\,dt - \sin(0)}{0^2} = \frac{0}{0}$$

$$= \lim_{x\to 0}\frac{e^{-x^2/2} - \cos(x)}{2x} = \frac{e^{-0} - \cos(0)}{2(0)} = \frac{1-1}{0} = \frac{0}{0}$$

$$= \lim_{x\to 0}\frac{e^{-x^2/2}\left(-\frac{2x}{2}\right) + \sin(x)}{2} = \frac{e^{-0}(0) + \sin(0)}{2} = \frac{0}{2} = \boxed{0}$$

2. (a) $u = \ln x \Rightarrow \frac{du}{dx} = \frac{1}{x}$

$$\int \frac{1}{x(\ln x)^3}\,dx = \int \frac{du}{dx}\,\frac{1}{u^3}\,dx = \int \frac{1}{u^3}\,du = \frac{1}{-2}u^{-2} + C$$

$$= -\frac{1}{2u^2} + C = -\frac{1}{2(\ln x)^2} + C$$

(b) $u = e^x \Rightarrow \frac{du}{dx} = e^x$ $\quad x = \ln\pi \Rightarrow u = e^x = e^{\ln\pi} = \pi$, $x = \ln\pi - \ln 2 \Rightarrow u = \frac{\pi}{2}$

$$\int_{\ln\pi - \ln 2}^{\ln\pi} e^x \sin(e^x)\,dx = \int_{u=\frac{\pi}{2}}^{u=\pi}\frac{du}{dx}\sin(u)\,dx = \int_{\pi/2}^{\pi}\sin(u)\,du$$

$$= -\cos(u)\Big|_{\pi/2}^{\pi} = -\cos(\pi) + \cos\left(\tfrac{\pi}{2}\right) = \boxed{1}$$

4. (a) $2 = x^3 + \sinh x + 2 \Rightarrow x = 0 \Rightarrow g(2) = f^{-1}(2) = \boxed{0}$

(b) $f'(x) = 3x^2 + \cosh(x) \Rightarrow g'(2) = \frac{1}{f'(f^{-1}(2))} = \frac{1}{f'(0)} = \frac{1}{3(0^2) + \cosh(0)} = \boxed{1}$

5. (a) TRUE (b) TRUE (c) False (d) FALSE