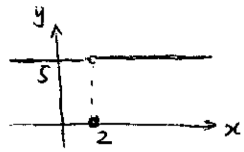
Math21, spring 2008

Solutions to Midterm 1

1 (a) $\frac{x^2-9}{x+3}$ is not defined at $x=-3$, but $x-3$ is.

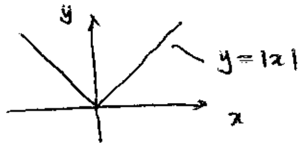
(b) $\lim_{x\to 2} f(x) = 5$ does not say anything about the value of $f(2)$.

For example



$\lim_{x\to 2} f(x) = 5$ but $f(2)=0$

(c)



$$\frac{d}{dx}|x| = \begin{cases} 1 & x>0 \\ -1 & x<0 \end{cases}$$

$f(x)=|x|$ is not differentiable at $x=0$.

(d) $y=|x|$ is continuous at $x=0$, but not differentiable at $x=0$

(e) $\lim_{x\to 0} \sin(x) = \sin(0) = 0$ too, so $\lim_{x\to 0} \frac{\sin(x)}{x}$ may still exists

even $\lim_{x\to 0} x = 0$ on the bottom. In fact $\lim_{x\to 0} \frac{\sin(x)}{x} = 1$.

2 (a) $\lim_{x\to 2} \frac{x^2+x-6}{x-2} = \lim_{x\to 2} \frac{(x+3)(x-2)}{x-2} = \lim_{x\to 2} x+3 = 2+3 = \boxed{5}$

(b) $-1 \le \sin\frac{1}{x} \le 1$ for all x $\Rightarrow$ $-x^2 \le x^2 \sin\frac{1}{x} \le x^2$ for all x

$\Rightarrow \lim_{x\to 0}(-x^2) \le \lim_{x\to 0} x^2 \sin\frac{1}{x} \le \lim_{x\to 0} x^2$

$\Rightarrow 0 \le \lim_{x\to 0} x^2 \sin\frac{1}{x} \le 0 \Rightarrow \lim_{x\to 0} x^2 \sin\frac{1}{x} = \boxed{0}$

(c)
$$\frac{|x-2|}{x-2} = \begin{cases} \frac{x-2}{x-2} = 1, & x>2 \\ \frac{-(x-2)}{x-2} = -1, & x<2 \end{cases}$$

$\Rightarrow \lim_{x\to 2^+} \frac{|x-2|}{x-2} = \lim_{x\to 2^+} 1 = 1$, $\lim_{x\to 2^-} \frac{|x-2|}{x-2} = \lim_{x\to 2^-}(-1) = -1$

$\Rightarrow \lim_{x\to 2} \frac{|x-2|}{x-2}$ $\boxed{\text{DNE}}$ since $1 \neq -1$.

3 (a) $$g'(x) = \lim_{h\to 0} \frac{g(x+h)-g(x)}{h} = \lim_{h\to 0} \frac{\frac{1}{1-(x+h)} - \frac{1}{1-x}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{(1-x)-[1-(x+h)]}{[1-(x+h)](1-x)}}{h} = \lim_{h\to 0} \frac{\frac{\cancel{1}-\cancel{x}-\cancel{1}+\cancel{x}+h}{[1-(x+h)](1-x)}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{\cancel{h}}{[1-(x+h)](1-x)}}{\cancel{h}} = \lim_{h\to 0} \frac{1}{[1-(x+h)](1-x)} = \frac{1}{(1-x)(1-x)}$$

$$= \boxed{\frac{1}{(1-x)^2}}$$

(b) $$y' = (14x-1)\sin(x) + (7x^2 - x + 1)\cos(x)$$

(c) $$g'(x) = -\sin\sqrt{x} \cdot \frac{1}{2\sqrt{x}} = -\frac{\sin\sqrt{x}}{2\sqrt{x}}$$

(d) $$y = 5x^2 + \frac{1}{x} \Rightarrow y' = 10x - \frac{1}{x^2}$$

at $x=-1$, $y'\big|_{x=-1} = -10 - 1 = -11$ — slope

$x=-1 \Rightarrow y\big|_{x=-1} = 5(-1)^2 + \frac{1}{-1} = 5-1 = 4$ — pt $(-1, 4)$

$\Rightarrow$ tangent line $\boxed{y-4 = -11(x+1)}$, or $\boxed{y = -11x - 7}$