UC Merced: Math 21 –Final Exam – May 17, 2008

On the front of your bluebook print (1) your name, (2) your student ID number, (3) your discussion section number and instructor's name (Lei or Yatskar) and (4) a grading table. Show all work in your bluebook and BOX IN YOUR FINAL ANSWERS where appropriate. A correct answer with no supporting work may receive no credit while an incorrect answer with some correct work may receive partial credit. Textbooks, class notes, calculators and crib sheets are not permitted. There are a total of eleven problems on both sides of this paper and a total of 150 points. Please start each of the eleven problems on a new page. You have 3 hours to complete the exam.

Some potential useful information:

$$\frac{d\arcsin(t)}{dt} = \frac{1}{\sqrt{1-t^2}}; \frac{d\arccos(t)}{dt} = \frac{-1}{\sqrt{1-t^2}}; \frac{d\arctan(t)}{dt} = \frac{1}{1+t^2};$$

For certain conditions, the following is true: $(f^{-1})'(a) = \frac{1}{f'(f^{-1}(a))}$

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1. (20 points: 2 each) Answer the following Always True or False. Only your final answer

a) $\lim_{x\to\infty} \frac{\sin x}{x} = \lim_{x\to\infty} \frac{\cos x}{1}$

b) If f(x) is differentiable at x=a, then $\lim_{x\to a} f(x) = f(a)$

c) If f'(x) >0 on interval [a,b] then f(b) > f(a)

d) If $\lim_{x\to 4} f(x) = 5$ and $\lim_{x\to 4} g(x) = 6$ then $\lim_{x\to 4}(g(x) - f(x)) = 1$

e) If one - to - one function f(x) has a vertical asymptote then $f^{-1}(x)$ has a horizontal asymptote.

f) $e^{\ln(-2)} = -2$

g) $f(x) = x^3$ has a local maximum at x=0

h) $\frac{d}{dx}\int_a^2 \frac{1}{t^2}dt = \frac{1}{2^2}$

i) $\int_{-2}^{2} \cosh(t)dt = 2\int_0^2 \cosh(t)dt$

j) Antiderivative of g(x) = |x| doesn't exist on [-1,1] since $\frac{dg}{dx}$ is not continuous at x=0

2) (15 points) Find the requested limits, if they exist. If they do not exist, explain.

a) (5 pts) $\lim_{x\to 0} \frac{x-1}{x^2(x+5)}$

b) (5 pts) $\lim_{x\to 0} \frac{1-\sqrt{1-x^2}}{x}$

c) (5 pts) $\lim_{x\to -\infty} x\cdot e^x$

3) (15 points) Calculate derivative $\frac{dy}{dt}$

a) (7 points) $y = \frac{10}{\cos(5t)}$

b) (8 points) $y = t\sqrt{1+t^2}$

4) (10 points) Use the derivative formula $\frac{d\arcsin(t)}{dt} = \frac{1}{\sqrt{1-t^2}}$ to calculate $\frac{d\sin(t)}{dt}$.

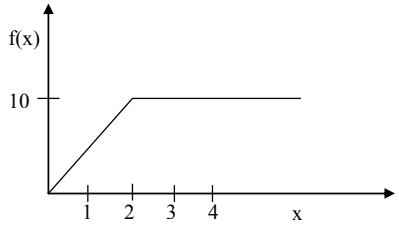
(*Hint: You may need* $\cos^2(t) + \sin^2(t) = 1$ *to simplify the answer)*

5) (10 points) Express the quantity as a single logarithm.

$\ln k + f \ln z - a \ln x$

6) (10 points) Find the equation of the tangent line to $x^2 + y^2 = 1$ at x=.5, y = .5 ?

7) (10 points) Find the $\int_1^3 f(x)dx$ from the diagram below.



8) (15 points) The rate at which a certain fast food restaurant sells burgers during a single day is given by $r(t) = 15t - t^2$ (burgers/hour) where $0 \le t \le 10$ hrs since the opening time at 10 am.

a) (7 points) What is the busiest time of the day?

b) (8 points) How many burgers do they sell in a day?

9) (15 points) Evaluate the following integrals:

a) (7 points) $\int_1^2 \frac{5+u^2}{u^3} du$

b) (8 points) $\int_1^2 e^{\sin\theta} \cos\theta d\theta$

10) (15 points) A spherical balloon is being inflated. Find the rate of increase of the surface area $S = 4\pi r^2$ with respect to the time when r = 10 cm and dr/dt = 1 cm/min

11) (15 points) The population of aphids on a rose plant is modeled by the equation $P(t)=P_0\exp(kt)$. In 3 days the population grew from 800 to 1400. How long will it take for population to double?