UC Merced: MATH 21 — Exam #1 — 20 February 2009

On the front of your bluebook print (1) your name, (2) your student ID number, (3) your discussion section number and instructor's name (Sprague, Lei, or Crona) and (4) a grading table. Show all work in your bluebook and BOX IN YOUR FINAL ANSWERS where appropriate. A correct answer with no supporting work may receive no credit while an incorrect answer with some correct work may receive partial credit. Textbooks, class notes, calculators and crib sheets are not permitted. There are a total of four problems and 50 points. Please start each of the four problems on a new page. You have 50 minutes to complete the exam.

Next Week's Quiz will be based on 2(a)-(c) and 4(a)-(c); keep your exam!

1. (10 points total; 2 points each) Answer the following **Always** True (T) or False (F) and short answer questions. Only your final answers will be graded on these problems.

   (a) (T or F) If $\lim\limits_{x \to 5} f(x) = \infty$ and $\lim\limits_{x \to 5} g(x) = -\infty$, then $\lim\limits_{x \to 5}[f(x) + g(x)] = 0$

   (b) (T or F) The following piecewise-defined function is differentiable at all $\theta \neq 0$ but is not differentiable at $\theta = 0$:
   $$g(\theta) = \begin{cases} \theta^2, & \theta < 0, \\ \theta^3, & \theta \geq 0. \end{cases}$$

   (c) (T or F) If $\lim\limits_{x \to 82} b(x)$ exists, then $b(x)$ must be continuous at $x = 82$.

   (d) (T or F) $\lim\limits_{x \to 0} x \sin\left(\dfrac{1}{x}\right)$ is undefined because $\lim\limits_{x \to 0} \dfrac{1}{x}$ is undefined.

   (e) (short answer) If the function $P(t)$ describes the number of people living in California in year t, what are the units of $P'(t)$?

2. (15 points; 5 points each) Answer the following unrelated questions that include material on limits and/or continuity

   (a) If it exists, determine the limit $\lim\limits_{x \to 3} \dfrac{x^2 - x - 6}{x - 3}$. If it does not exist, explain why.

   (b) If it exists, determine the limit $\lim\limits_{x \to -2} \dfrac{2 - |x|}{2 + x}$. If it does not exist, explain why.

   (c) Write down a function $f(x)$ that satisfies the following: (i) continuous everywhere except $x = 5$, (ii) $\lim\limits_{x \to 5^-} f(x) = +\infty$, (iii) $\lim\limits_{x \to 5^+} f(x) = -\infty$, (iv) $f(0) = 0$. (Note: write down an actual function; no sketch required)

3. (10 points) Using the definition of the derivative, calculate $\dfrac{dm}{dx}$ where $m(x) = \dfrac{1}{\sqrt{x+1}}$.

4. (15 points) Answer the following unrelated derivative questions. You are welcome to use differentiation shortcuts for (a) and (b).

   (a) (5 points) Determine $\dfrac{dw}{dt}$, where $w(t) = (\sqrt{t} + \dfrac{1}{\sqrt{t}})\cos(t^2) + 5$

   (b) (5 points) Use your knowledge of $\frac{d}{d\theta}\sin(\theta)$ and $\frac{d}{d\theta}\cos(\theta)$ to show that $\dfrac{d}{d\theta}\tan(\theta) = [\sec(\theta)]^2$.

   (c) (5 points) Trace the graph of the following function $f(x)$ (on backside of sheet) into your green/blue exam book. Sketch the graph of $\dfrac{df}{dx}$ below your copy.