MATH 21, SPRING 2009 EXAM 1 SOL'N

1)(a) F (b) F (c) F (d) F (e) $\frac{\text{PEOPLE}}{\text{YEAR}}$

2)(a) $\lim_{x\to 3} \frac{x^2-x-6}{x-3} = \lim_{x\to 3} \frac{(x+2)(x-3)}{x-3} = \lim_{x\to 3} (x+2) = \boxed{5}$

(b) $\lim_{x\to -2} \frac{2-|x|}{2+x} = \lim_{x\to -2} \frac{2-(-x)}{2+x} = \lim_{x\to -2} 1 = \boxed{1}$

(c) $\boxed{f(x) = \frac{-1}{x-5} - \frac{1}{5}}$

3) $\frac{dm}{dx} = \lim_{h\to 0} \frac{1}{h}\left[\frac{1}{\sqrt{x+h+1}} - \frac{1}{\sqrt{x+1}}\right] = \lim_{h\to 0}\left[\frac{\sqrt{x+1} - \sqrt{x+h+1}}{\sqrt{x+h+1}\sqrt{x+1}}\right]$

$= \lim_{h\to 0} \frac{1}{h}\left[\frac{\sqrt{x+1} - \sqrt{x+h+1}}{\sqrt{x+h+1}\sqrt{x+1}}\right]\left[\frac{\sqrt{x+1} + \sqrt{x+h+1}}{\sqrt{x+1} + \sqrt{x+h+1}}\right]$

$= \lim_{h\to 0} \frac{1}{h}\left[\frac{(x+1) - (x+h+1)}{\sqrt{x+h+1}\sqrt{x+1}\left(\sqrt{x+1} + \sqrt{x+h+1}\right)}\right]$

$= \lim_{h\to 0} \frac{1}{h}\left[\frac{-h}{\sqrt{x+h+1}\sqrt{x+1}\left(\sqrt{x+1} + \sqrt{x+h+1}\right)}\right]$

$= \frac{-1}{(x+1)\left(\sqrt{x+1} + \sqrt{x+1}\right)} = \frac{-1}{2(x+1)\sqrt{x+1}} \Rightarrow \boxed{\frac{dm}{dx} = \frac{-1}{2}(x+1)^{-3/2}}$

4)(a) $\frac{dw}{dt} = \frac{d}{dt}\left(t^{1/2} + t^{-1/2}\right)\cos(t^2) + \left(t^{1/2} + t^{-1/2}\right)\frac{d}{dt}\cos(t^2)$

$= \left(\frac{1}{2}t^{-\frac{1}{2}} - \frac{1}{2}t^{-3/2}\right)\cos(t^2) - \left(t^{1/2} + t^{-1/2}\right)\sin(t^2)\frac{d}{dt}(t^2)$

$\boxed{\frac{dw}{dt} = \left(\frac{1}{2}t^{-\frac{1}{2}} - \frac{1}{2}t^{-3/2}\right)\cos(t^2) - \left(t^{1/2} + t^{-1/2}\right)\sin(t^2)\,2t}$

(b) $\frac{d}{d\theta}\tan\theta = \frac{d}{d\theta}\frac{\sin\theta}{\cos\theta} = \frac{\cos(\theta)\frac{d}{d\theta}\sin\theta - \sin\theta\frac{d}{d\theta}\cos\theta}{(\cos\theta)^2}$

$= \frac{(\cos\theta)^2 + (\sin\theta)^2}{(\cos\theta)^2} = \frac{1}{(\cos\theta)^2} = \boxed{[\sec\theta]^2}$

(c) GRAPH OF $f'(x)$