Math 21 Spring 2009 Exam 2 solutions

1(a). [Worksheet #8: 7] $k \geq 2$

(b). [Worksheet #7: 4] No

(c). $V'(10) \leq 0$ since $V(t)$ is a decreasing function

(d). False

(e). [Homework §4.2: 21(a)] True

2. [Compare with Worksheet #6: 3]

$$2xy^2 + x^2(2y)\frac{dy}{dx} + 3y^2\frac{dy}{dx} = 0$$

At $(1,1)$: $2 + 2\frac{dy}{dx} + 3\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{2}{5} \Rightarrow$ tangent line at $(1,1)$ is $\boxed{y - 1 = -\frac{2}{5}(x-1)}$

3. [Compare with Homework §4.9: 60]

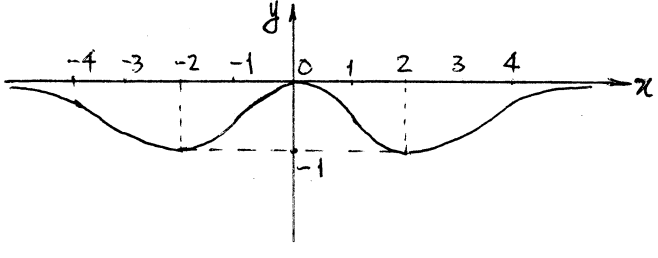
$v(t) = \sin(t) - \cos(t) + C$, $3 = v(0) = \sin(0) - \cos(0) + C = 0 - 1 + C$

$\Rightarrow C = 4 \Rightarrow v(t) = \sin(t) - \cos(t) + 4$

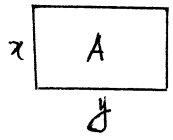
$S(t) = -\cos(t) - \sin(t) + 4t + D$ $\quad 1 = S(0) = -\cos(0) - \sin(0) + 4(0) + D = -1 + D$

$\Rightarrow D = 2 \Rightarrow \boxed{S(t) = -\cos(t) - \sin(t) + 4t + 2}$

4. [compare with Homework §4.4: 52]



5. [Homework and Quiz §4.7: 16(a)]



x = length, y = width, A = area, P = perimeter

$P = 2x + 2y$ minimize P.

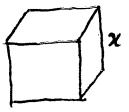
$A = xy \Rightarrow y = \frac{A}{x} \Rightarrow P = 2x + \frac{2A}{x}$

$\frac{dP}{dx} = 2 - \frac{2A}{x^2} = 0 \Rightarrow 2x^2 = 2A \Rightarrow x^2 = A = x = \sqrt{A}$

$\frac{d^2P}{dx^2} = \frac{4A}{x^3} > 0$ for all positive $x \Rightarrow P = 2x + \frac{2A}{x}$ is concave up everywhere $\Rightarrow x = \sqrt{A}$ is a minimum of P.

$x = \sqrt{A} \Rightarrow y = \frac{A}{\sqrt{A}} = \sqrt{A} \Rightarrow$ The rectangle is a square.

6.



x = length of edge A = surface area

$A = 6x^2$ $\frac{dA}{dt} = -10\ cm^2/min$, $x = 5\ cm$, $\frac{dx}{dt} = ?$

$\frac{dA}{dt} = 12x\frac{dx}{dt} \Rightarrow -10\ cm^2/min = 12(5\ cm)\frac{dx}{dt}$

$\Rightarrow \frac{dx}{dt} = \frac{-10\ cm^2/min}{60\ cm} = \boxed{-\frac{1}{6}\ cm/min}$

[Compare with Homework and Quiz §3.8 : 12]