Math 22 Unit Exam 2

Instructions. Read each problem carefully and follow all of its instructions. For each of the problems, please write the best solution you can as clearly as you can in your blue book. For short answer and true/false questions, write clearly your answer and any additional explanations needed.

Problems.

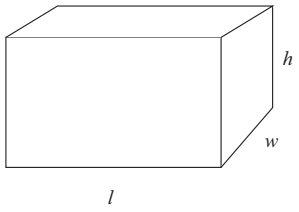
1. (10 points) I can give you \$100 now or \$100 after you graduate from UC Merced. Ignore inflation. Explain why one of these options is better than the other.

2. (10 points) Use the comparison test to explain whether or not
$$\int_0^\infty \cos^2(x)\mathrm{e}^{-x}\mathrm{d}x$$
converges.

3. (10 points) Find the x and y coordinates for center of mass of a plate of constant density δ covering the region bounded above by the parabola $y = 4 - x^2$ and below by the x-axis.

4. For each statements below (a)-(e), say whether they are true or false. If the statement is false, revise the statement to make it true.

   For the following three questions (a)-(c), consider the fish tank drawn below with dimensions l ft $\times$ w ft $\times$ h ft which is filled with water.



   (a) (2 points) The force on the bottom of the tank depends quadradically on the depth h.

   (b) (2 points) The force on the $l \times h$ side of the tank depends quadradically on the depth h.

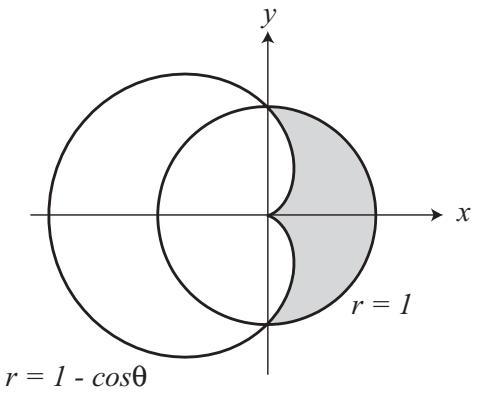
   (c) (2 points) The work to pump the water out from top is equal to the force on the $w \times h$ side times l.

   (d) (2 points) The integral $\int_0^\infty (x+4)^{-1}\mathrm{d}x$ converges.

   (e) (2 points) If \$$B$ denotes the future value and \$$P$ denotes the present value, then $P \geq B$ independent of how interest is compounded.

5. (10 points) The region bounded by the curve $y = x^2 + 1$ and the line $y = -x + 3$ from $-2 \leq x \leq 1$ is revolved about the x-axis to generate a solid. Using the method of slicing, derive a definite integral for the volume of this solid (you do not need to compute this integral).

6. (10 points) Find the area of the shaded region shown in the figure below that lies inside of the circle $r = 1$ and outside of the limaçon $r = 1 - \cos\theta$.



You may find the following trigonometric identities useful in this computation.

$$\cos^2\theta = \frac{1}{2}[1 + \cos(2\theta)], \qquad \sin^2\theta = \frac{1}{2}[1 - \cos(2\theta)]$$
$$\sin(2\theta) = 2\sin(\theta)\cos(\theta), \qquad \cos(2\theta) = 1 - 2\sin^2(\theta)$$

7. (10 points) Find the arclength of the curve defined by the parametric equations:

$$x = \cos^3 t, \quad y = \sin^3 t, \quad 0 \leq t \leq \pi/4.$$