Math 22 - Calculus II

Midterm 1 - Solutions.

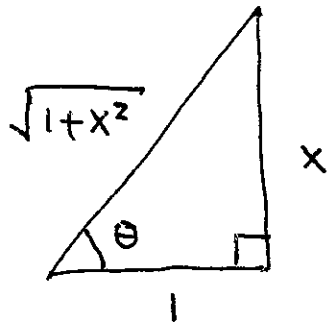
1. $$\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx = 2\int e^u\,du \qquad u=\sqrt{x},\ du=\frac{1}{2\sqrt{x}}\,dx$$
$$= 2e^u + C$$
$$= 2e^{\sqrt{x}} + C.$$

2. $$\int_0^\pi x\cos x\,dx = x\sin x\Big|_0^\pi - \int_0^\pi \sin x\,dx$$
$$= x\sin x\Big|_0^\pi + \cos x\Big|_0^\pi$$
$$= \pi\sin\pi - 0\sin 0 + \cos\pi - \cos 0$$
$$= -2$$

3. $$\int \frac{1}{(1+x^2)^{5/2}}\,dx = \int \frac{1}{(1+\tan^2\theta)^{5/2}}\sec^2\theta\,d\theta \qquad x=\tan\theta,\ dx=\sec^2\theta\,d\theta$$
$$= \int \frac{\sec^2\theta}{(\sec^2\theta)^{5/2}}\,d\theta$$
$$= \int \cos^3\theta\,d\theta$$
$$= \frac{1}{3}\cos^2 x\sin x + \frac{2}{3}\int \cos x\,dx$$
$$= \frac{1}{3}\cos^2 x\sin x + \frac{2}{3}\sin x + C$$

$$= \frac{1}{3} \cdot \frac{1}{1+x^2} \cdot \frac{x}{\sqrt{1+x^2}} + \frac{2}{3} \frac{x}{\sqrt{1+x^2}} + C$$

$$= \frac{2x^3 + 3x}{3(1+x^2)^{3/2}} + C$$



4.

$$\frac{x^2 - x}{x^3 + x^2 + x + 1} = \frac{x^2 - x}{(1+x)(1+x^2)} = \frac{A}{1+x} + \frac{Bx + C}{1+x^2}.$$

Cross-multiplying, we get

$$x^2 - x = A(1+x^2) + (Bx + C)(1+x)$$

i.e.

$$x^2 - x = (A+B)x^2 + (B+C)x + A + C$$

Equating coefficients, we get

$$\begin{aligned} A + B &= 1 \\ B + C &= -1 \\ A + C &= 0 \end{aligned}$$

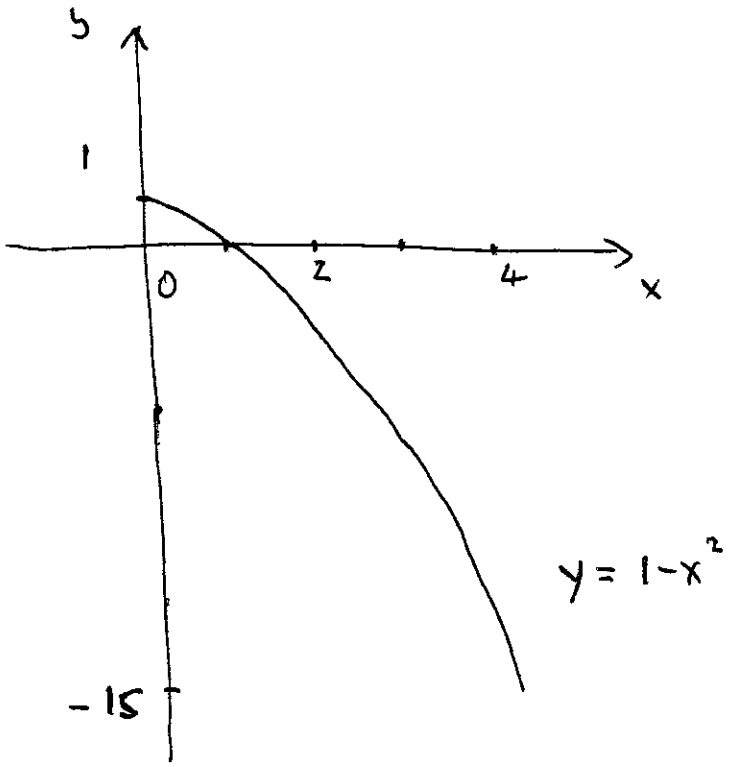
Solving, $A = 1$, $B = 0$, and $C = -1$.

Hence,

$$\int \frac{x^2 - x}{x^3 + x^2 + x + 1} \, dx = \int \left(\frac{1}{1+x} - \frac{1}{1+x^2} \right) dx$$

$$= \ln|1+x| - \tan^{-1} x + C.$$

5.



$f(x) = 1-x^2$

$\Delta x = \frac{4-0}{2} = 2$

(i) $MID(2) = f(1)\Delta x + f(3)\Delta x$

$= 0 \cdot 2 + (-8) \cdot 2$

$= -16$

$TRAP(2) = \frac{LEFT(2) + RIGHT(2)}{2}$

$= \frac{f(0)\Delta x + f(2)\Delta x + f(2)\Delta x + f(4)\Delta x}{2}$

$= \frac{1 \cdot 2 + 2 \cdot (-3) \cdot 2 + (-15) \cdot 2}{2}$

$= -20.$

(ii) $f'(x) = -2x \quad f''(x) = -2 < 0.$

The function is concave down and so $MID(2)$ is an over-estimate and $TRAP(2)$ is an under-estimate.