Instructions. Attempt all questions. Answers must be justified in order to gain full credit. Calculators are not permitted.

1. (10 points) By making a trigonometric substitution, find

$$\int \frac{1}{x^2\sqrt{1+x^2}}\,dx$$

2. (8 points) Use the method of partial fractions to find

$$\int \frac{2x}{x^3 - x^2 + x - 1}\,dx$$

3. (10 points) The region bounded by $y = e^x$, the x-axis, and the lines $x = 1$ and $x = 2$ is rotated about the line $y = -2$. Find the volume of the resulting solid.

4. (i) (5 points) Graph the polar curve $r = \cos 2\theta$.

   (ii) (5 points) Find the area of one loop of the curve.

   Note: You may find the trigonometric identity $\cos^2 x = (1 + \cos 2x)/2$ useful.

5. (10 points) Find the total mass of the triangular region below which has density $\delta(x) = 1 + x$ g/cm^2.
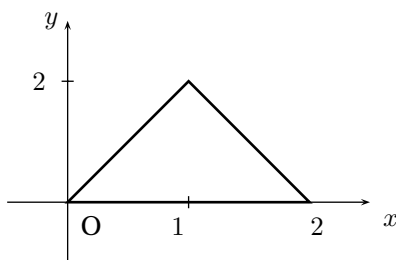
6. (10 points) An anchor weighing 150 lb in water is attached to a chain weighing 4 lb/ft in water. Find the work done to haul the anchor and chain to the surface of the water from a depth of 50 ft.

7. (8 points) Use the integral test to determine if the series $\displaystyle\sum_{n=1}^{\infty} \frac{3}{n^2 + 4}$ converges.

8. (5 points) Determine whether or not the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^3}$ is absolutely convergent.

9. (6 points) Use the alternating series test to show that the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{e^n}$ converges.

10. (10 points) Find the radius of convergence and the interval of convergence for the power series $\displaystyle\sum_{n=1}^{\infty} \frac{n^2 x^{2n}}{2^{2n}}$.

11. (5 points) Find the first four nonzero terms of the Talyor series for $f(x) = \ln x$ about $x = 1$.

12. (8 points) Find the Lagrange error bound when $f(x) = \dfrac{1}{\sqrt{1+x}}$ is approximated by its third-degree Talyor polynomial about $x = 0$ at $x = 0.1$.