Math 22: Unit 1 Exam

Spring Semester 2006

Instructions. Read each problem carefully and follow all of its instructions. For each of the problems below, write a clear and concise solution in your blue book. For any short answer questions, write clearly your answer and any additional explanation that is needed.

1. (5 points) Explain why you must add a constant when computing an indefinite integral, but not when computing a definite integral.

2. (5 points) Explain the difference between a proper rational function and and improper rational function.

3. (10 points) Evaluate exactly the integral

$$\int_0^2 x^7 e^{x^4} \mathrm{d}x.$$

   (Hint: Start with the substitution $w = x^4$.)

4. (10 points) Find

$$\int \frac{x^2}{\sqrt{9-4x^2}} \mathrm{d}x.$$

   You may or may not find the following trigonometric identities useful in this computation.

$$\cos^2\theta = \frac{1}{2}[1+\cos(2\theta)], \qquad \sin^2\theta = \frac{1}{2}[1-\cos(2\theta)]$$
$$\sin(2\theta) = 2\sin(\theta)\cos(\theta), \qquad \cos(2\theta) = 1-2\sin^2(\theta)$$

5. (10 points) According to Formula III-16 in the back of your textbook,

$$\int p(x)\cos(x)\mathrm{d}x = p(x)\sin(x) + p'(x)\cos(x) - p''(x)\sin(x) - \cdots$$

   with the signs in the series alternating in pairs $(++--++--\ldots)$. Use this formula to determine the value of a so that

$$\int_0^\pi (x^2+ax+1)\cos(x)\mathrm{d}x = 7\pi.$$

6. (10 points) Use partial fractions to find

$$\int \frac{x+1}{x^3-x^2} \mathrm{d}x.$$