Name:_____

SID:_____

Section:_____
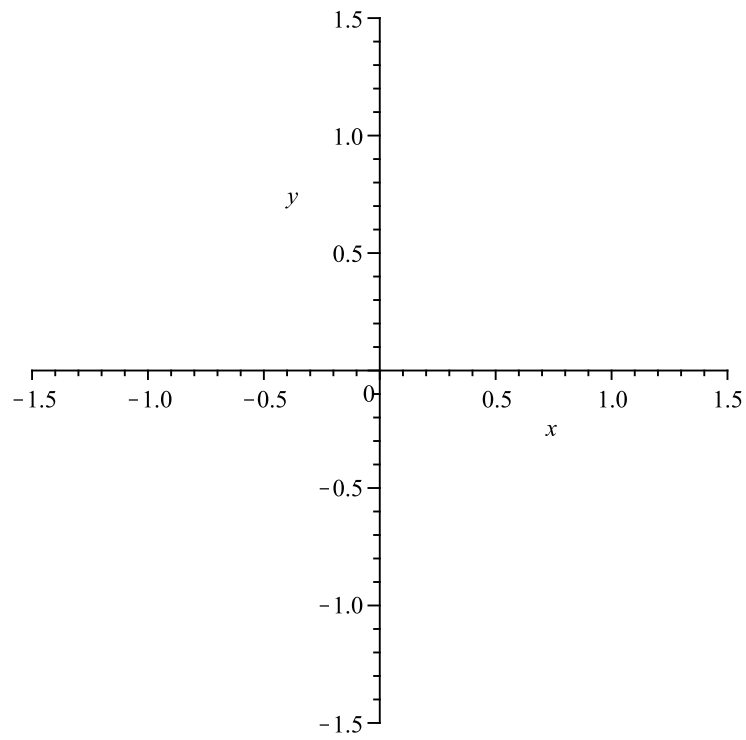
Final Exam -Math 022-F '07

The exam is 3 hours long. No calculators or notes are permitted. Show your work. You do not need to simplify your answers.

	Problem	Score out of 10
	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
Extra Credit	9	
	Total Out of 90	

1. It is a fact that $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$.

   How well does $\displaystyle\sum_{n=1}^{20} \frac{1}{n^2}$ approximate $\dfrac{\pi^2}{6}$?

2. Sketch the polar curve $r = \sin(2\theta)$ on the coordinate axes given below. Find the area of one "leaf".

3. Find the area of the finite region bounded by the curves $y = x^2$, $y = (x - 2)^2$, and $y = 0$.

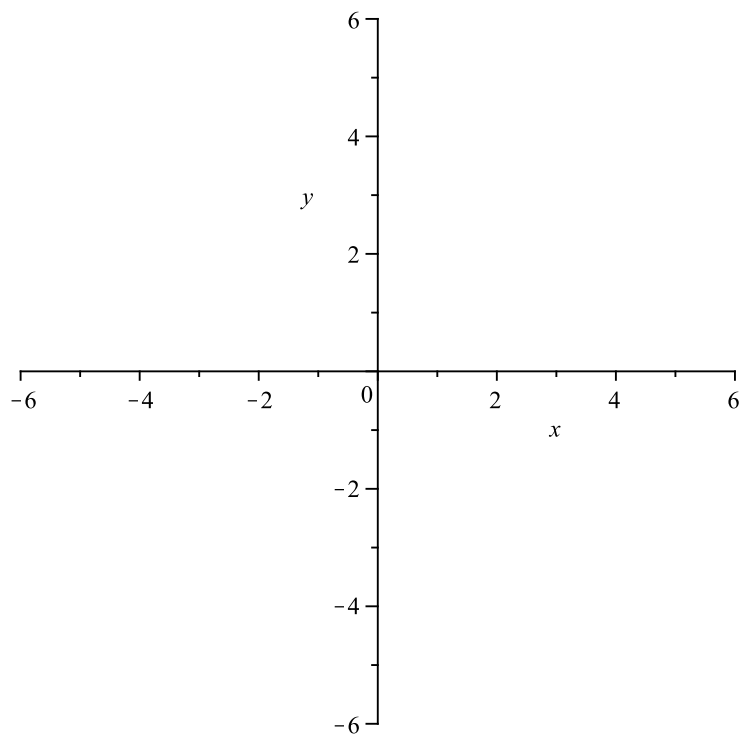*For the follow two problems, determine whether the series converges. Explain your reasoning for both.*

4.

$$\sum_{n=1}^{\infty} e^{-\frac{n^2}{2}}$$

5.

$$\sum_{n=1}^{\infty} \left(\frac{1}{n} \right)^n$$

6. In the coordinate axes given below, carefully sketch the conic section given by

$$\frac{x^2}{4} - y^2 = 1.$$

7. Find the partial fraction decomposition for $\dfrac{x^2 - x + 1}{(x-1)^3}$.

8. Find the volume of the solid resulting from rotating the region bounded by the following curves around the y-axis.

$$x = 1, \, x = 2, \, y = 0, \, y = 1/x$$

9. Explain why the following integral is improper. Then compute its value (if it has one).

$$\int_0^1 x^2 \ln x\, dx$$

10. (Extra Credit 5 points)

The series $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$ converges to a finite number s. Find s.

Hint: What is $s - \frac{1}{2}s$?