Midterm 1, Math 22, Fall 2008, 10/6/08

1) Find the volume of a solid when the region bounded by $y = x^2$ *and* $y = \sqrt{x}$ revolves around the x-axis (10 pts)

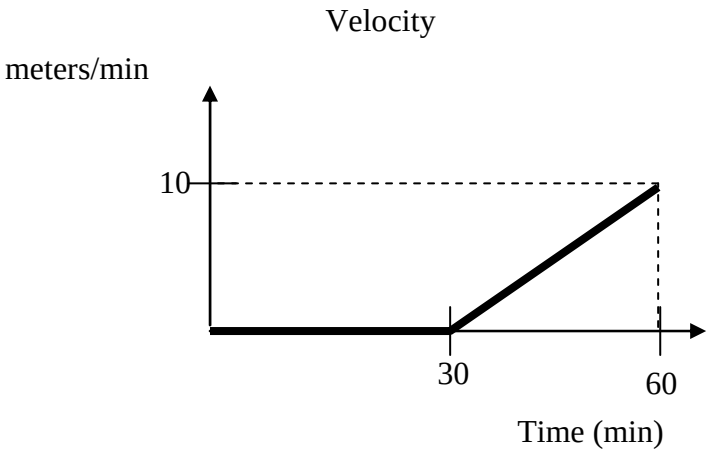
2) Find the number b so that the average of the function $y = bx - x^2$ on the interval [0,2] is $\frac{4}{3}$ (10 pts)

Evaluate the following integrals: Answer must be in terms of x

3) $\int \frac{1}{x^2\sqrt{x^2-9}}dx$ (10 pts)

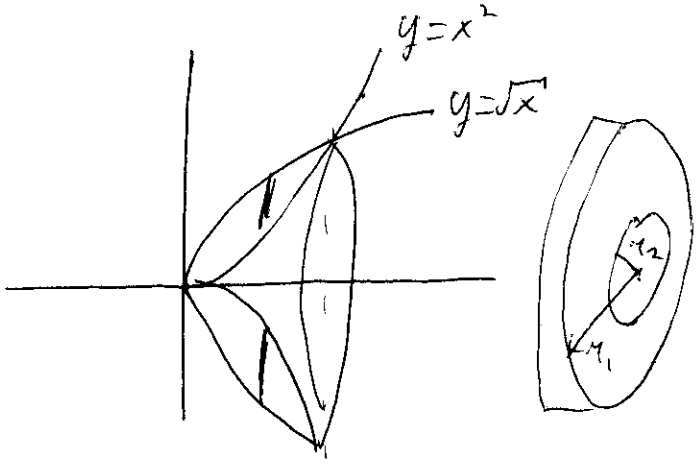
4) $\int \frac{2x+5}{x^2+5x+6}dx$ (10 pts)

5) Velocity of the car traveling in a straight line is shown in the diagram. Find the total displacement of the car after 50 min (10 pts). What is happening to the car during the first 30 min? (2 pts)



MATH 22, MIDTERM 1

(1)



Washer Method

$V = \pi \int r_1^2 - r_2^2 \, dx$ $\qquad r_1 = \sqrt{x}$, $r_2 = x^2$

Find intersection points:

$x^2 = \sqrt{x}$ $\qquad x^4 = x$ $\qquad x(x^3-1)=0$

Curves intersect at $x = 0, 1$

$$V = \pi \int_0^1 x - x^4 \, dx = \pi \left(\frac{x^2}{2} - \frac{x^5}{5} \right) \Big|_0^1$$

$$\boxed{V = \frac{3\pi}{10}}$$

(2) $y = 6x - x^2$ $\qquad [0, 2]$

$$y_{AV} = \frac{1}{2-0} \int_0^2 6x - x^2 \, dx = \frac{1}{2} \left(\frac{6x^2}{2} - \frac{x^3}{3} \right) \Big|_0^2$$

$$y_{AV} = \frac{1}{2} \left(26 - \frac{8}{3} \right) = 6 - \frac{4}{3} = 4/3$$

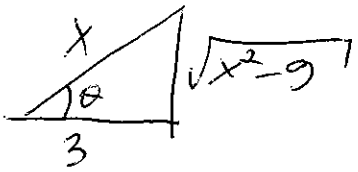
$$\boxed{6 = \frac{8}{3}}$$

(3) $\int \frac{1}{x^2\sqrt{x^2-9}}\,dx$ $\quad x = 3\sec\theta$, $\sqrt{x^2-9} = 3\tan\theta$

$dx = 3\sec\theta\tan\theta\,d\theta$

$$\int \frac{3\sec\theta\tan\theta\,d\theta}{9\sec^2\theta\ 3\tan\theta} = \int \frac{d\theta}{\sec\theta} = \int \cos\theta\,d\theta$$

$= \sin\theta$ convert to x



$\sin\theta = \frac{\sqrt{x^2-9}}{x}$

$$\boxed{\int \frac{dx}{x^2\sqrt{x^2-9}} = \frac{\sqrt{x^2-9}}{9x} + C}$$

(4) $\int \frac{2x+5}{x^2+5x+6}\,dx$

$$\frac{2x+5}{(x+3)(x+2)} = \frac{A}{x+3} + \frac{B}{x+2}$$

$2x+5 = A(x+2) + B(x+3) = (A+B)x + (2A+3B)$

$A+B=2 \quad *2 = 2A+2B=4$

$2A+3B=5 \quad = \quad 2A+3B=5$

Subtract $\quad -B = -1 \quad \boxed{B=1}$

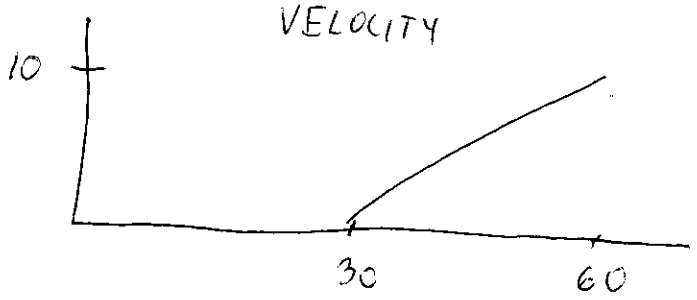
$A+B=2$ or $\boxed{A=1}$

$$\int \frac{2x+5}{x^2+5x+6}\,dx = \ln|x+3| + \ln|x+2| + C$$

Easier way $\quad u = x^2+5x+6 \quad du = (2x+5)dx$

$$\int \frac{du}{u} = \ln|u| = \ln|x^2+5x+6| + C$$

(5)



slope $= \frac{10}{30} = \frac{1}{3}$

$$V(t) = \begin{cases} 0 & 0 < t < 30 \\ \frac{1}{3}(t-30) & t \geq 30 \end{cases}$$

Displacement $= \int_0^{50} V(t)\,dt$

$$= \frac{1}{3}\int_{30}^{50} (t-30)\,dt = \frac{1}{3}\left(\frac{t^2}{2} - 30t\right)\Big|_{30}^{50}$$

$$= \frac{1}{3}\left(\frac{2500}{2} - 1500 - \frac{900}{2} + 900\right) = 66.7 \text{ meters}$$

$\frac{200}{3}$