Midterm 3, Math 22, Fall 2008, 12/08/08

Instructions: Write your name and section number. Draw grading table on the cover. Read each problem carefully and follow all of its instructions. For each of the problems below, write a clear and concise solution in your blue book. Solutions must be simplified as much as possible, no full credit for partially completed problems. **Blue books with torn or missing pages will not be accepted !**

1. (10 pts) Answer the following Always True (T) or False (F). Only your final answers will be graded on these problems. Unless specified $\sum$ refers to infinite sum. (1 pt each)
   a. Alternating series are monotonic
   b. If $\sum a_n$ is a convergent series then $\sum \frac{1}{a_n}$ is also a convergent series.
   c. If $\sum a_n$ is a divergent series with positive terms then $\sum (-1)^n a_n$ also divergent.
   d. Integral test can be used to calculate the exact sum of infinite series.
   e. Series $\sum \sin(n)$ converges
   f. If $\sum a_n$ and $\sum b_n$ are series with positive terms $a_n > b_n$ and $\sum b_n$ diverges then $\sum a_n$ also diverges.
   g. Ratio test can be used to prove that $\sum n^{-1}$ diverges.
   h. Series $\sum (-1)^n / n$ converges absolutely
   i. Remainder can be used to estimate the error of a partial sum
   j. If $\sum a_n$ is a convergent series then $\lim_{n\to\infty} a_n = 0$

2. (10 pts) Amount of money deposited in the bank account every month is given by the formula $a_n = 100\left(\frac{2^n + 4^n}{5^n}\right)$ where n is the month number starting with n=1. If initial account balance is \$0, find the balance after a long time.

3. (10 pts) The terms of a series are defined recursively by the equation $a_1 = 2 \quad a_{n+1} = \frac{n}{n^2+2} a_n$. Determine whether $\sum_{n=1}^{\infty} a_n$ converges or diverges.

4. (10 pts) Determine whether the series $\sum_{n=1}^{\infty} \frac{n}{n^3 - 1}$ is convergent or divergent.

5. (10 pts) Show that $\sum_{n=1}^{\infty} \frac{5^n}{n!}$ converges.