Math 22 Midterm 3 Solutions

1) True or false (1 pt. each)

a. $\boxed{F}$ - "Monotonic" means always increasing or always decreasing. Alternating series don't do this; they alternate above + below zero.

b. $\boxed{F}$ - If Σa_n converges then $\lim_{n\to\infty} a_n = 0$. But $\lim_{n\to\infty} \frac{1}{a_n} \neq 0$ so $\Sigma \frac{1}{a_n}$ diverges.

c. $\boxed{F}$ - Any conditionally convergent series contradicts this statement.

d. $\boxed{F}$ - The integral test states that $\int_1^\infty f(x)dx$ and $\sum_{n=1}^\infty a_n$ behave the same if $a_n = f(n)$, but they are not equivalent.

e. $\boxed{F}$ - The test for divergence states when $\lim_{n\to\infty} a_n \neq 0$ then Σa_n diverges.

f. $\boxed{T}$ - This is a complete and correct application of the comparison test.

g. $\boxed{F}$ - Ratio test is inappropriate to use for polynomial fraction expressions. The limit of the correct ratio will be 1, so the test is inconclusive.

h. $\boxed{F}$ - This series converges conditionally, not absolutely.

i. $\boxed{T}$ - Since $S = S_N + R_N$ the error in S_N is accurately described by R_N.

j. $\boxed{T}$ - This is simple fact.

---

2) $a_n = 100\left(\frac{2^n + 4^n}{5^n}\right)$ $n = 1, 2, 3, \ldots$ (+1 start)

$$S = \sum_{n=1}^{\infty} 100\left(\frac{2^n+4^n}{5^n}\right) = 100\left[\sum_{n=1}^{\infty}\left(\frac{2}{5}\right)^n + \sum_{n=1}^{\infty}\left(\frac{4}{5}\right)^n\right] = \underbrace{100\left[\sum_{n=1}^{\infty}\frac{2}{5}\left(\frac{2}{5}\right)^{n-1} + \sum_{n=1}^{\infty}\frac{4}{5}\left(\frac{4}{5}\right)^{n-1}\right]}_{+3}$$

Check $r_1 = \frac{2}{5} < 1$ (+2) $r_2 = \frac{4}{5} < 1$ (+2)

$$= 100\left[\left(\frac{2/5}{1-2/5}\right) + \left(\frac{4/5}{1-4/5}\right)\right] = \boxed{\$\ \frac{1400}{3}} \text{ (+2)}$$

---

3) $a_1 = 2$ $a_{n+1} = \frac{n}{n^2+2} a_n$ Find if $\sum_{n=1}^{\infty} a_n$ converges or diverges. (+1 start)

Use ratio test, (+2)

$|a_n| = ?$ $|a_{n+1}| = \frac{n}{n^2+2} a_n$

$$\underbrace{L = \lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{\frac{n}{n^2+2}a_n}{a_n}\right|}_{+3} = \lim_{n\to\infty}\left|\frac{n}{n^2+2}\right|$$

$$= \lim_{n\to\infty}\left|\frac{1}{n+2/n}\right| = 0 \text{ (+2)}$$

False assumption
$a_n = \frac{n}{n^2+2}$

Check $L = 0 < 1$ (+1)

By ratio test

$\boxed{\Sigma a_n \text{ converges}}$

(+1)

Math 22 Midterm 3 Solutions

4) Find if $\sum_{n=2}^{\infty} \frac{n}{n^3-1}$ converges. (+1 start)

Using comparison test

$a_n = \frac{n}{n^3-1}$ $\quad b_n = \frac{2}{n^2}$ (+2)

(+3) Check $a_n \le b_n$

$\frac{n}{n^3-1} \le \frac{2}{n^3}$

$0 \le n^3 - 2$

$\sum b_n$ converges by p-series $p = 2 > 1$ (+2)

$\boxed{\sum a_n \text{ converges}}$ (+2)

Using limit comparison test

$a_n = \frac{n}{n^3-1}$ $\quad b_n = \frac{1}{n^2}$ (+2)

$\sum b_n$ converges by p-series $p = 2 > 1$ (+2)

(+3) Check $L = \lim_{n\to\infty} \frac{a_n}{b_n} \neq 0$

$\lim_{n\to\infty} \left(\frac{n}{n^3-1}\right)\left(\frac{n^2}{1}\right)$

$= \lim_{n\to\infty} \left(\frac{n^3}{n^3-1}\right) = 1$

$L = 1 \neq 0$ $\quad \sum a_n$ & $\sum b_n$ behave the same

$\boxed{\sum a_n \text{ converges}}$ (+2)

Note: The itegral test could also work, but involves showing $\int_2^{\infty} \frac{x}{x^3-1}\,dx$ converges.

5) Find if $\sum_{n=1}^{\infty} \frac{5^n}{n!}$ converges. (+1 start)

Use ratio test. (+2 evidence)

$|a_n| = \frac{5^n}{n!}$ $\quad |a_{n+1}| = \frac{5^{n+1}}{(n+1)!}$

$$L = \overbrace{\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{5^{n+1}}{(n+1)!} \cdot \frac{n!}{5^n}\right|}^{+3} = \lim_{n\to\infty} \left|\frac{5}{n+1}\right| = 0 \quad (+2)$$

Check $L = 0 < 1$ (+1)

$\boxed{\sum a_n \text{ converges}}$ (+1)