Name:______________________________

Vector Calculus **Exam 3** **November 1st**

There are 6 problems and 144 points total. The point value of each question is indicated. *Read each question carefully!*

1. (24 points.) Compute the following iterated integrals

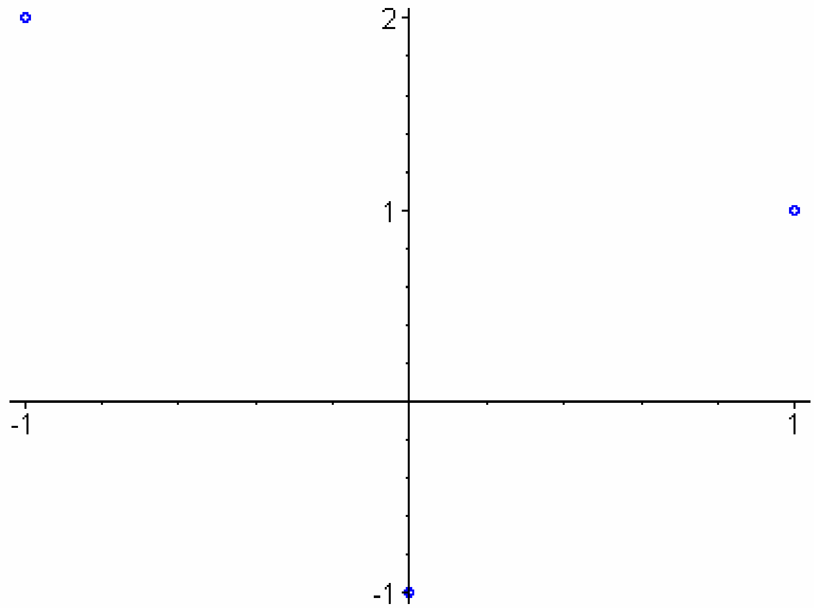
a) $\int_0^2 \int_0^3 x^2 + y^2 dydx$

b) $\int_0^1 \int_0^1 e^{x+y} dxdy$

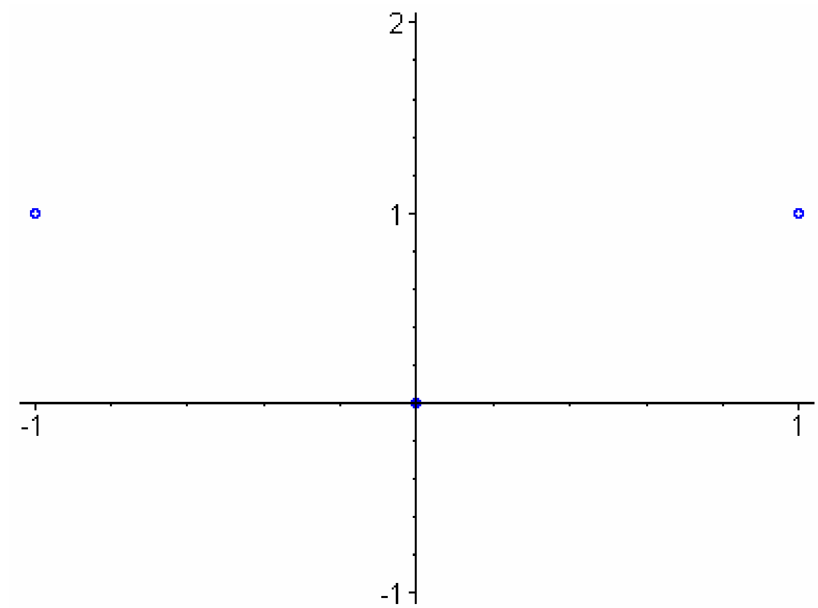
c) $\int_0^1 \int_{-r^2}^{1-r^2} r d\theta dr$

d) $\int_p^q \int_c^d \int_a^b 1 dxdydz$

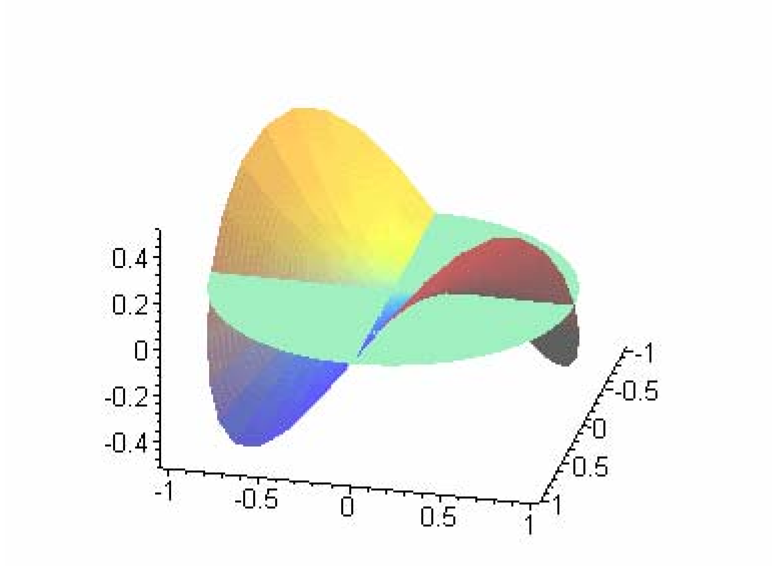
2. (24 points.) Compute the regression line with the form $a + bx$ for the points (-1,2), (0,-1), (1,1) using least squares and draw the regression line in the figure.



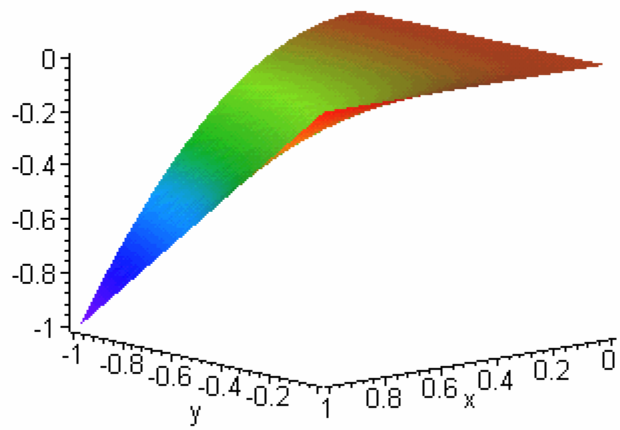
3. (24 points.) Compute the regression line with the form $a+bx$ for the points (-1,1), (0,0), (1,1) using least squares and draw the regression line in the figure.



4. (24 points.) Compute the double integral of the function $f(x, y) = xy$ over the region $R = \{(x, y) : x^2 + y^2 \leq 1\}$

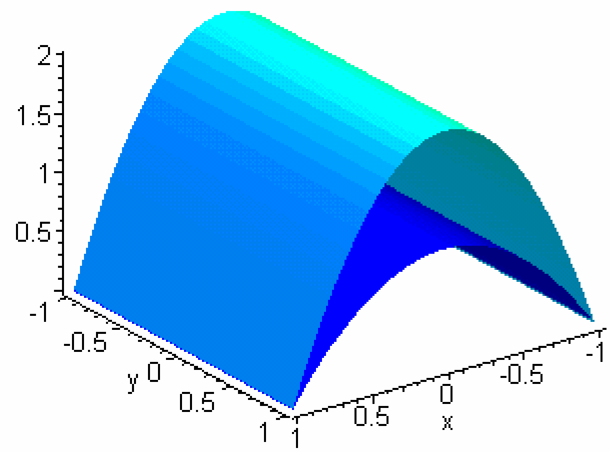


5. (24 points.) Find the *volume* between the graph of the function $f(x,y)=x^2y$ and the $xy-$plane over the region $R=\{(x,y):0\le x\le 1,-1\le y\le 0\}$



6. The region W is bounded on the top by $z = 2 - 2x^2$, on the bottom by $z = 1 - x^2$, on the front by $y = 1$, and on the back $y = -1$.

a) (8 points) Express the volume of W as three iterated integrals.



b) (16 points) Compute the volume. of W using any method.

Extra credit *Do not work on any of these until you have finished the rest of the exam!*

A) (6 points) Compute $\int_0^1 \int_y^1 e^{-x^2}\, dxdy$

B) (4 points) Notice in problems 2) and 3) that the mean of the x coordinates, $\bar{x}$, and the mean of the y coordinates, $\bar{y}$, gives us a point $(\bar{x}, \bar{y})$ which lies on the regression line. This is a general fact which is closely related to the fact that the axis of rotation for a rigid body must pass through its center of mass. Use this fact to create a ***new*** set of three points whose regression line has the same y-intercept as the point set in problem 2)

C) (2 points) Is the volume in problem 5) larger or smaller than the volume in problem 6)?