Name:_____

Vector Calculus **Exam 4** **November 22nd**

There are 7 problems and 144 points total. The point value of each question is indicated. *Read each question carefully!*

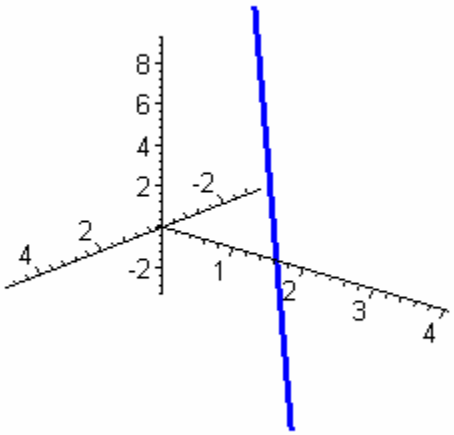
1. (20 points.) Compute the following iterated integrals

a) $\int_0^3 \int_0^y y^2 dxdy$

b) $\int_0^1 \int_{\sqrt[4]{y}}^1 5\sqrt{1+x^5}\, dxdy$

2. (18 points.) Let $R = \{(x, y) : x^2 + y^2 \leq 4\pi^2\}$. Compute $\int_R \sin(x^2 + y^2)dA$ using polar coordinates.

3. (18 points.) Find a formula for the line which passes through the point (1,2,3) in the direction of the vector (2,1-3)



4. (28 points.) Find the velocity and acceleration along the following curves:

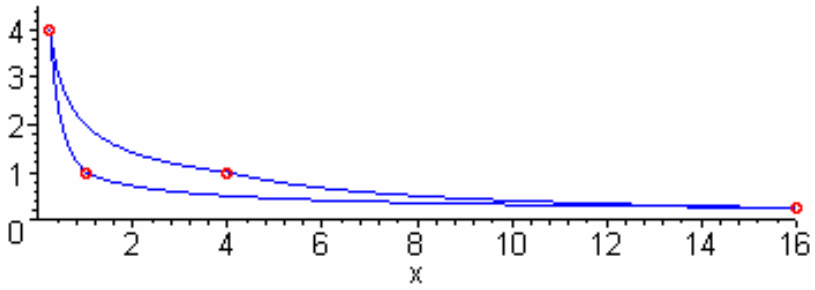
a) $\vec{r}(t) = (3\cos(t), 2\sin(t), -t)$

b) $\vec{r}(t) = (\cos(t^2), \sin(t^2), t^2)$

c) $\vec{r}(t) = \left(\frac{e^{2t} + e^{-2t}}{2}, \sin(t), \frac{e^{2t} - e^{-2t}}{2} \right)$

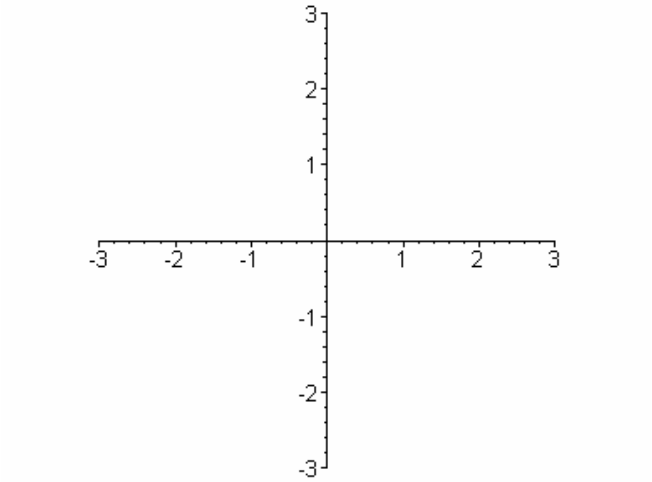
d) $\vec{r}(t) = \left(e^{t}, e^{-t/2} \cos\left(\frac{\sqrt{3}}{2} t \right), e^{-t/2} \sin\left(\frac{\sqrt{3}}{2} t \right) \right)$

5. (20 points.) Let $R = \{(x, y) : 1 \le xy \le 4 \text{ and } 1 \le xy^2 \le 4\}$ Use the change of variables $s = xy$ $t = xy^2$ to compute $\int_R xy^2 dA$



6.

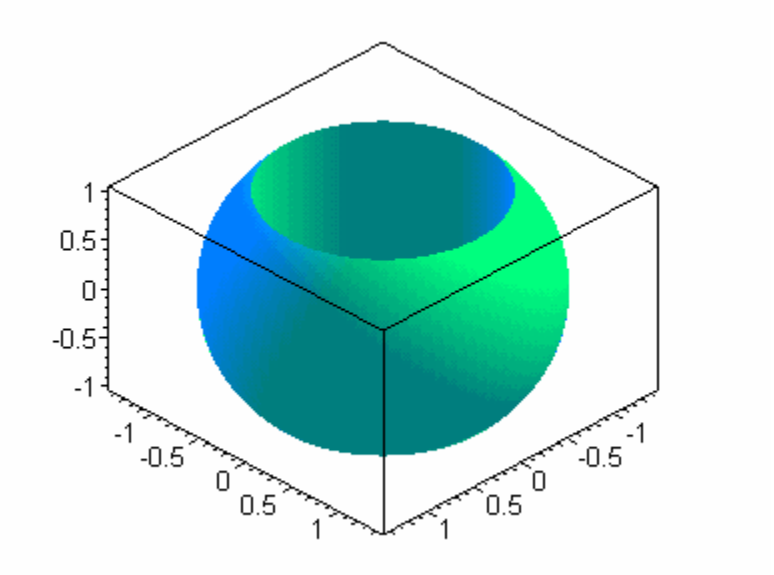
a) (7 points) Sketch the vector field $\vec{F}(x,y) = (y,x)$



b) (10 points) Check that $\vec{r}(t) = \left(e^t + e^{-t}, e^t - e^{-t}\right)$ is a flow line of the vector field $\vec{F}(x,y) = (y,x)$

c) (3 points) Draw the flow line $\vec{r}(t) = \left(e^t + e^{-t}, e^t - e^{-t}\right)$ in the vector field $\vec{F}(x,y) = (y,x)$

7. (20 points.) Let $W = \{(x, y, z) : x^2 + y^2 \geq 1 \text{ and } x^2 + y^2 + z^2 \leq 2\}$ Compute $\int_W x^2 + y^2 dV$



Extra credit *Do not work on any of these until you have finished the rest of the exam!*

A) (4 points) Choose two curves from problem 4) and sketch them.

B) (6 points) Compute $\int_{-\infty}^{\infty} e^{-x^2}\,dx$