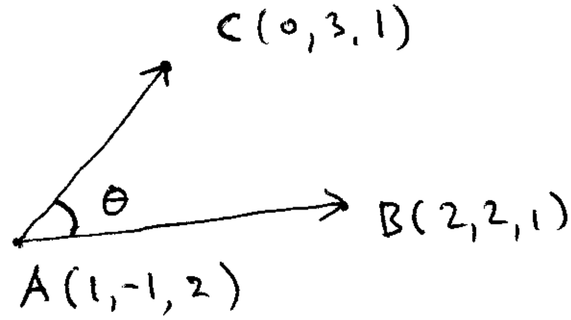
Math 23 - Calculus III

Midterm 1 - Solutions.

1.



$$\vec{AC} = -\vec{i} + 4\vec{j} - \vec{k}, \quad \vec{AB} = \vec{i} + 3\vec{j} - \vec{k}$$

$$\vec{AC} \cdot \vec{AB} = \|\vec{AC}\| \, \|\vec{AB}\| \cos\theta$$

$$(-1)\cdot 1 + 4\cdot 3 + (-1)(-1) = \sqrt{(-1)^2 + 4^2 + (-1)^2}\sqrt{1^2 + 3^2 + (-1)^2} \cos\theta$$

$$\cos\theta = \frac{4}{\sqrt{22}}$$

$$\angle BAC = \theta = \cos^{-1}\frac{4}{\sqrt{22}}$$

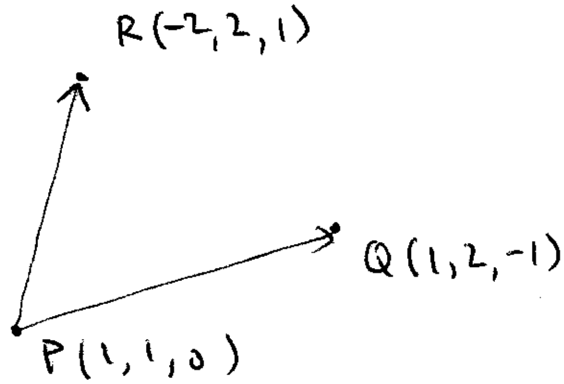
2.

$$\left[(\vec{b}\cdot\vec{c})\vec{a} - (\vec{a}\cdot\vec{c})\vec{b}\right]\cdot\vec{c}$$

$$= (\vec{b}\cdot\vec{c})(\vec{a}\cdot\vec{c}) - (\vec{a}\cdot\vec{c})(\vec{b}\cdot\vec{c})$$

$$= 0.$$

3.



$$\vec{PR} = -3\vec{i} + \vec{j} + \vec{k}$$

$$\vec{PQ} = \vec{j} - \vec{k}$$

(i) $$\overrightarrow{PR} \times \overrightarrow{PQ} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -3 & 1 & 1 \\ 0 & 1 & -1 \end{vmatrix} = -2\vec{i} - 3\vec{j} - 3\vec{k}.$$

Area of $\Delta PQR = \frac{1}{2} \| \overrightarrow{PR} \times \overrightarrow{PQ} \| = \frac{1}{2}\sqrt{(-2)^2 + (-3)^2 + (-3)^2} = \frac{1}{2}\sqrt{22}$

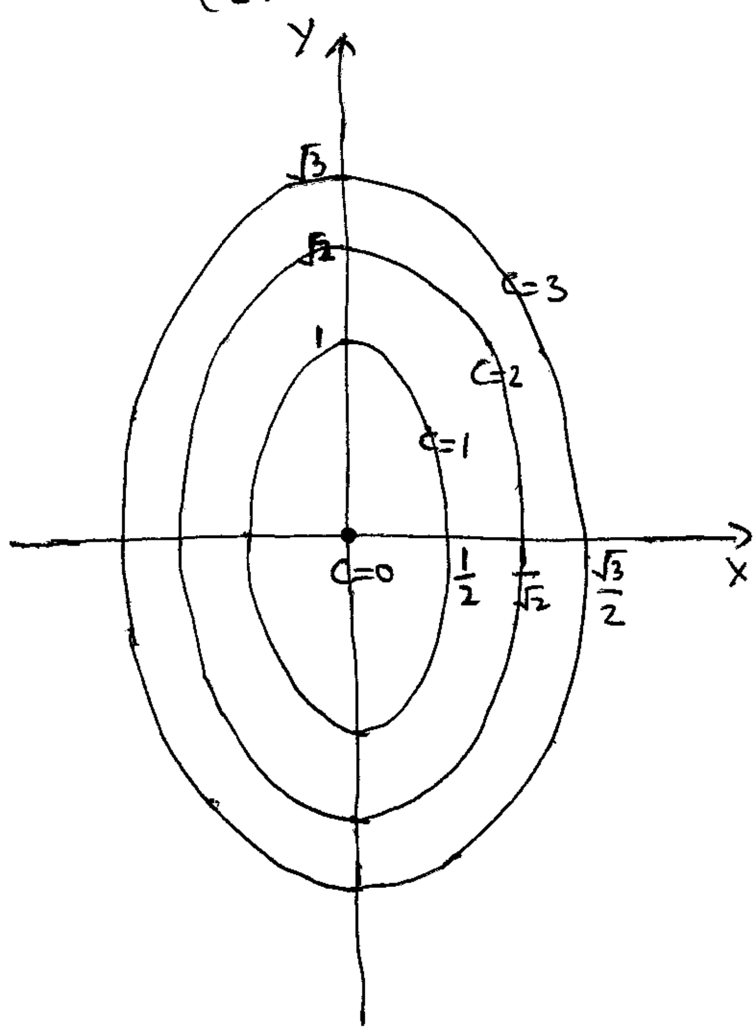
(ii) An equation of the plane is

$$(-2\vec{i} - 3\vec{j} - 3\vec{k}) \cdot ((x-1)\vec{i} + (y-1)\vec{j} + z\vec{k}) = 0$$

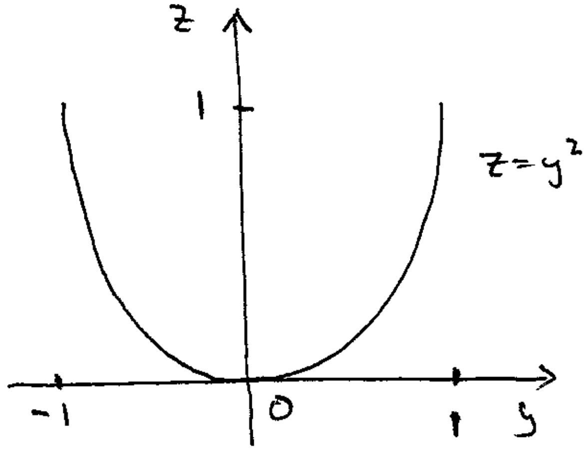
i.e. $$2x + 3y + 3z - 5 = 0.$$

4 (i) $$4x^2 + y^2 = c$$

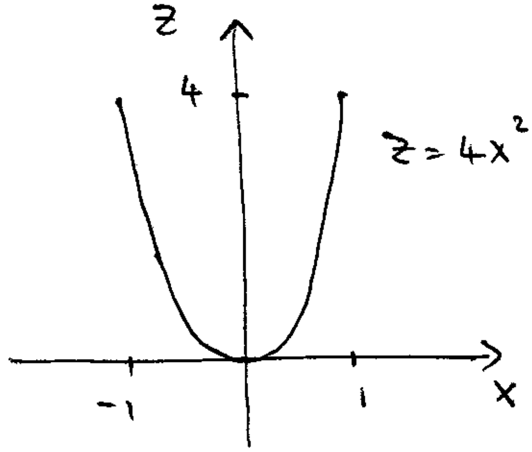
or $$\frac{x^2}{\left(\frac{\sqrt{c}}{2}\right)^2} + \frac{y^2}{(\sqrt{c})^2} = 1$$



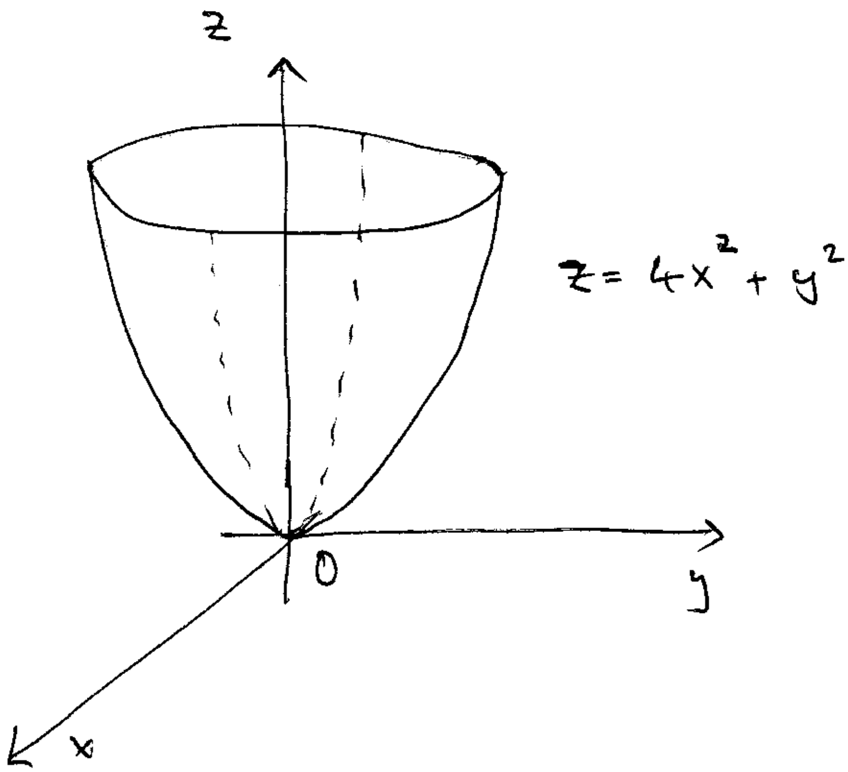
(ii) $x = 0$ $\qquad z = f(0, y) = y^2$



$y = 0$ $\qquad z = f(x, 0) = 4x^2$



(iii)



5. $$(x+1)^2 + (y-1)^2 + z^2 = 16$$

$$(x+1)^2 + (y-1)^2 + z^2 - 13 = 3$$

$$f(x,y,z) = (x+1)^2 + (y-1)^2 + z^2 - 13.$$

6. Approaching along the lines $y = mx$:

$$\lim_{x \to 0} f(x, mx) = \lim_{x \to 0} \frac{3x^4 . mx}{x^6 + (mx)^3}$$

$$= \lim_{x \to 0} \cancel{3m} \frac{3mx^2}{x^3 + m^3}$$

$$= 0$$

For $x = 0$:

$$\lim_{y \to 0} f(0, y) = \lim_{y \to 0} \frac{3 \cdot 0^4 \cdot y}{0^6 + y^3}$$

$$= 0.$$

No conclusion.

Along the parabolas $y = kx^2$:

$$\lim_{x \to 0} f(x, kx^2) = \lim_{x \to 0} \frac{3x^4 . kx^2}{x^6 + (kx^2)^3}$$

$$= \lim_{x \to 0} \frac{3k}{1+k^3}.$$

$$= \frac{3k}{1+k^3}.$$

Since these limits depend on the path, $\lim\limits_{(x,y) \to (0,0)} \frac{3x^4 y}{x^6 + y^3}$ does not exist.

7. We require that $\lim_{(x,y)\to(3,y)} f(x,y)$ exist for all y.

That is,

$$\lim_{(x,y)\to(3^-,y)} f(x,y) = \lim_{(x,y)\to(3^+,y)} f(x,y)$$

That is,

$$c + y = 5 - y$$

$$\text{or} \qquad c = 5 - 2y$$

There is no single value of c that works for all y and so there is no single value of c that makes f continuous at $(3,y)$ for all y.