Duration: 50 minutes
Instructions: Answer all questions, without the use of notes, books or calculators. Partial credit will be awarded for correct work, unless otherwise specified. The total number of points is 100.

1. (20 pts) Given the function $f(x, y) = 9x^2 + y^2 - 1$
   (a) Draw at least 2 cross-sections of $f(x, y)$ with x fixed.
   (b) Draw at least 2 contours.
   (c) **SKETCH** the surface in a manner consistent with what you found above.

2. (18 pts) A plane $p(x, y)$ is parallel to the vectors $\vec{v}_1 = \vec{i} - \vec{j} - \vec{k}$ and $\vec{v}_2 = 2\vec{i} + \vec{j}$.
   (a) Find the normal of this plane.
   (b) Find the equation of $p(x, y)$ if the plane goes through the point $(0, 2, -1)$.

3. (20 pts) Above the point $(0, 1)$ in the xy-plane, the plan tangent to a function $f(x, y)$ is $p(x, y) = 5 + x - 3y$.
   a) What is $f(0, 1)$?
   b) What is the gradient of $f(x, y)$ at $(0, 1)$?
   c) What is the directional derivative of $f(x, y)$ at $(0, 1)$ in the direction $\vec{v} = -\vec{i} - \vec{j}$?
   d) If x and y are functions of time $x(t) = \sin t$ and $y(t) = e^{2t}$, compute $\frac{df}{dt}$ at $t = 0$.

4. (18 pts) Consider the function $f(x, y) = x^3 - 3x + y^2 - y$.
   (a) Find and classify all the critical points of $f(x, y)$.
   (b) Does this function have a global minimum over $D = \{\text{all } x \geq 1 \text{ and all } y \geq 1\}$? If so, find it, if not explain why.

5. (24 pts) Answer the following questions in no more than two lines of text. No computations are required.
   (a) What is the normal of the plane tangent to the surface $g(x, y, z) = 0$ at a point (x_0, y_0, z_0) on the surface?
   (b) If you are told that the point (x_0, y_0) maximizes $f(x, y)$ subject to $g(x, y) = 0$, what can you say about the partial derivatives or gradients of these functions at (x_0, y_0)?
   (c) At a given point, the gradient of $f(x, y)$ is $\nabla f = \vec{i} + \vec{j}$. In what direction would you have to move if you wanted to maintain a constant value of $f(x, y)$?
   (d) If $B(s, r)$ is the price of burritos, s the price of salsa and r the price of rice, what is the meaning of $\frac{\partial B}{\partial r}$?
   (e) If you know that $\lim_{x\to 0} f(x, mx) = \lim_{x\to 0} f(x, kx^2) = 2$, what can you conclude about the continuity of $f(x, y)$ at the origin?
   (f) If $\vec{a} \cdot (\vec{b} \times \vec{c}) = 0$, what can you conclude about the vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$?