Duration: 50 minutes
Instructions: Answer all questions, without the use of notes, books or calculators. Partial credit will be awarded for correct work, unless otherwise specified. The total number of points is 50.

1. (10 pts) Given the function $z^2 = x^2 + y^2 - 1$

   a) Draw cross-sections of $f(x, y)$ with x fixed.

   b) Draw at least 3 contours.

   c) Noting the symmetry in x and y, **SKETCH** the surface in a manner consistent with what you found above.

2. (9 pts) Given the vector $\vec{N} = \vec{i} - 2\vec{j} + 3\vec{k}$,

   a) Find the equation of the plane perpendicular to $\vec{N}$ and going through $p = (0, 2, 4)$.

   b) Decompose the vector $\vec{v} = \vec{j} - \vec{k}$ into two parts $\vec{a}$ and $\vec{b}$ such that $\vec{a}$ is parallel to $\vec{N}$, $\vec{b}$ is perpendicular to $\vec{N}$ and $\vec{v} = \vec{a} + \vec{b}$.

   c) Find a vector perpendicular to both $\vec{N}$ and $\vec{v}$.

3. (10 pts) Consider the function $f(x, y) = e^{2x} \sin y$.

   a) What is the tangent plane to $f(x, y)$ above $(0, \pi/4)$?

   b) In which direction does $f(x, y)$ decreases the fastest at $(0, \pi/4)$?

   c) What is the maximum rate of increase of $f(x, y)$ at that point?

   d) If x and y depend on another variable u: $x(u) = u^3$ and $y(u) = \pi/4 + 3u$, compute $\frac{df}{du}$ at $u = 0$.

4. (9 pts) Consider the function $f(x, y) = (x^2 - 4)(y^2 - 1)$.

   a) Find all the critical points of $f(x, y)$.

   b) Select a critical point and determine if it is a maximum, minimum or saddle-point.

   c) Over the domain $x \geq 1$ and $y \geq 2$, does this function have a global minimum? If so find it, if not, why not?.

5. (12 pts) Answer the following questions in no more than two lines of text (much less is actually needed if you are right on point). No computations are required.

   a) Is it possible to have a function which is differentiable but not continuous? How about a continuous function which is not differentiable?

   b) If you want to minimize $f(x, y)$ subject to $g(x, y) = c$, what is the Lagrangian function you should use?

   c) Describe in words the level surfaces of $g(x, y, z) = (x + 2y - 3z)^3$.

   d) Explain in words the meaning of the directional derivative of $f(x, y) = x/y$ at the point $(2, 3)$ in the direction $-\vec{i} - 2\vec{j}$.

   e) Give one possible use of the quadratic expansion of a function.

   f) How long is a vector which is the sum of two perpendicular unit vectors?