Duration: 50 minutes

Instructions: Answer all questions, without the use of books or calculators. You may have a half sheet of 8.5X11 paper with both sides filled out. Partial credit will be awarded for correct work. You may use the back of the pages of the exam should it be necessary, but please indicate in writing that you have done so. The total number of points is 100.

Problem	Score
1	
2	
3	
4	
5	
6	
Total	

1. (15 points) Determine whether the following limit exists. If it does not exist, find two paths leading to the origin which give two different limits.

$$\lim_{(x,y)\to(0,0)} \frac{xy}{x^2 + xy + y^2}$$

2. (20 points)

    (a) Find the partial derivatives of $z = \sqrt{x + e^{4y}}$ at $(x, y) = (3, 0)$.

    (b) Use linearization and your answer in part (a) to approximate $\sqrt{3.1 + e^{-0.2}}$.

3. (15 points) Let P be the plane tangent to the surface $xyz = 1$ at the point $(1, 2, 0.5)$. Find the three points where P intersects a coordinate axis.

4. (20 points)

   (a) Find all the critical points of $z = xy + \dfrac{1}{x} + \dfrac{1}{y}$.

   (b) Classify the critical points you found in (a).

5. (15 points) A box must have a fixed volume V_0. Suppose the top and bottom of the box cost 2 cents per square inch and the sides cost 1 cent per square inch. Find the proportions that the dimensions of the box must lie in relation to each other in order to minimize the cost.

6. (15 points) Compute the following double integral where D is the region bounded by the curves $y = 2$, $y = \sqrt{x}$, and $x = 0$.

$$\iint_D \frac{1}{y^3 + 1} \, dA$$