UC Merced: MATH 24 — Exam #2 — 26 October 2006

On the front of your bluebook print (1) your name, (2) your student ID number, (3) your instructor's name (Sprague) and (4) a grading table. Show all work in your bluebook and BOX IN YOUR FINAL ANSWERS where appropriate. A correct answer with no supporting work may receive no credit while an incorrect answer with some correct work may receive partial credit. Textbooks, class notes, and calculators are not permitted. You are permitted on 8.5×11 inch crib sheet. There are a total of five problems and a total of 100 points. Please start each of the five problems on a new page.

---

1. **(20 points)** Answer the following Always True or False. Only your final **boxed** answer will be graded on these problems, and you must write out the words TRUE or FALSE completely.

   (a) The vectors $\mathbf{u} = (1, 5, 2)^T$ and $\mathbf{v} = (2, 2, -6)^T$ are perpendicular.

   (b) $f(t)$ and $tf(t)$ are linearly independent $\forall f \in \mathcal{C}^1$ and $\forall t \in \mathbb{R}$.

   (c) The capacitor in an LRC circuit is analogous to the mass in a classical mechanical oscillator.

   (d) The vectors $\{(1, 3)^T, (2, 6)^T, (-3, -9)^T\}$ span $\mathbb{R}^2$.

   (e) If B is as given below, then $\det B = -10$.

$$B = \begin{bmatrix} 0 & 5 & 6 & 2 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 2 & 1 \\ 1 & 0 & 0 & 2 \end{bmatrix}$$

2. **(25 points total)**

   (a) **(5 points)** Do the following vectors constitute a basis for $\mathbb{R}^3$? Be sure to justify your answer.

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \mathbf{v}_2 = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} \mathbf{v}_3 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$$

   (b) **(10 points)** Consider the set $\mathbb{W} = \{f(x) | \frac{d^4 f}{dx^4} + bf = 0, \forall b \in \mathbb{R}\}$. Given that $\mathbb{W}$ is a subset of $\mathcal{C}^4$ (the set of all continuous functions with 4 continuous derivatives), determine if $\mathbb{W}$ constitutes a **subspace** of $\mathcal{C}^4$.

   (c) **(10 points)** What is the dimension of the subspace of $\mathbb{P}_3$ spanned by the subset $S = \{t, t-1, t^2+1\}$?

3. **(15 points)** Consider the ordinary differential equation for $u(x)$:

$$2\frac{d^2u}{dx^2} + 6\frac{du}{dx} + 2u = 0$$

   (a) **(5 points)** Find the characteristic equation and its roots.

   (b) **(5 points)** Write down the *general* solution of the differential equation.

   (c) **(5 points)** Write down the *unique* solution for $u(0) = 1$, $u'(0) = 0$.

4. **(35 points total)**

   (a) **(5 points)** Consider the following matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

   If the operation is defined, calculate the matrix-matrix product AA^T. If not, explain why not.

   (b) **(5 points)** Consider the following system of equations:

$$\begin{aligned} 2x + 4y &= 2 \\ 4x + 9y - 3z &= 8 \\ -2x - 7z &= 10 \end{aligned}$$

   Determine the matrix A, and vectors $\mathbf{x}$, $\mathbf{b}$, such that the above system can be written $A\mathbf{x} = \mathbf{b}$. *Do not solve the system of equations.*

   (c) **(15 points)** Employ Gauss elimination to solve the linear system $A\mathbf{x} = \mathbf{b}$, where

$$A = \begin{bmatrix} 2 & 0 & 1 \\ -2 & 2 & 1 \\ 2 & -4 & 1 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}.$$

   Clearly label your operations. Using only your *final* solution as justification, what can you say regarding the value of $\det A$.

   (d) **(10 points)** Consider the following matrix:

$$B = \begin{bmatrix} 1 & 2 \\ -1 & k \end{bmatrix}$$

   Before trying to calculate B^{-1}, determine the values of k for which B^{-1} exists. Then, calculate B^{-1} that is valid for all of these values of k.

5. **(5 points total)** Write down a mathematical model for the following statement: In some forest, the change in the number of rabbits at a given time is proportional to the square root of the number of rabbits in the forest at that time.