UC Merced: MATH 24 — Exam #3 — 30 November 2006

On the front of your bluebook print (1) your name, (2) your student ID number, (3) your instructor's name (Sprague) and (4) a grading table. Show all work in your bluebook and BOX IN YOUR FINAL ANSWERS where appropriate. A correct answer with no supporting work may receive no credit while an incorrect answer with some correct work may receive partial credit. Textbooks, class notes, and calculators are not permitted. You are permitted an 8.5×11 inch crib sheet. There are a total of five problems and a total of 100 points. Please start each of the five problems on a new page.

---

1. **(20 points)** Answer the following Always True or False. Only your final **boxed** answer will be graded on these problems, and you must write out the words TRUE or FALSE completely.

   (a) For a 2×2 eigenproblem, if the eigenvalues are distinct, then there are two linearly independent eigenvectors.

   (b) The phase portrait ($y'(t)$ vs. $y(t)$) for the differential equation $y''+y'+y=0$ is composed of closed circles centered at the origin.

   (c) Consider the differential equation for $y(t)$: $y''+y'=t^2+1$. $y_p(t)=At^2+Bt+C$ is a suitable guess for the particular solution where A, B, C are unknown constants.

   (d) $\mathbf{v}^T=[-4,1,0]$ is an eigenvector associated with the eigenvalue $\lambda=2$ for the matrix

   $$A = \begin{bmatrix} 3 & 4 & 6 \\ -\frac{1}{2} & 0 & -3 \\ 0 & 0 & 3 \end{bmatrix}$$

   (e) Leonhard Euler, the famous Swiss mathematician, was born in the year 1783.

2. **(30 points total)**

   (a) **(5 points)** Consider the second order differential equation governing $y(t)$.

   $$\frac{d^2y}{dt^2}+2\frac{dy}{dt}+y=0.$$

   Is the system considered over damped, critically damped, or under damped?

   (b) **(13 points)** Consider the second-order differential equation governing $y(t)$:

   $$\frac{d^2y}{dt^2}+4\frac{dy}{dt}+4y=te^{-t}$$

   Given that the homogeneous solution is $y_h(t)=c_1e^{-2t}+c_2te^{-2t}$, use the method of Undetermined Coefficients to determine a particular solution $y_p(t)$.

   (c) **(12 points)** Here, we consider Variation of Parameters for solution of a nonhomogeneous second-order differential equation, $y''+py'+qy=f(t)$, where p and q are constants. Given that the homogeneous solution is known ($y_h(t)=c_1y_1(t)+c_2y_2(t)$), assume that

   $$y_p(t)=v_1(t)y_1(t)+v_2(t)y_2(t)$$

   Assuming further that $v_1'(t)y_1(t)+v_2'(t)y_2(t)=0$, determine the two sets of equations in the two unknowns $v_1'(t)$, $v_2'(t)$. Clearly show your work; you do not have to solve for $v_1'(t)$, $v_2'(t)$.

3. **(5 points)** Consider the following matrix:

   $$B = \begin{bmatrix} 1 & 2 \\ -1 & k \end{bmatrix}$$

   Determine the values of k for which B^{-1} exists.

4. **(30 points total)**

   (a) **(10 points)** Determine the eigenvalues and eigenvectors for the matrix A, where

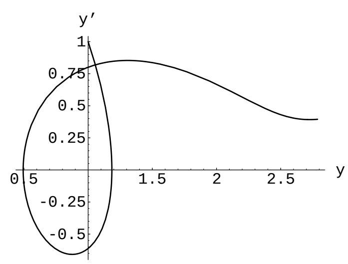
$$A = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix}$$

   (b) **(10 points)** Consider the system of differential equations $\mathbf{x}' = A\mathbf{x}$ for $\mathbf{x}^T(t) = [x_1(t), x_2(t)]$ where A is defined in part (a). On $x_2(t)$ vs. $x_1(t)$ axes, carefully sketch the eigenvectors and several trajectories on and off the eigenvectors to sufficiently describe the solution space. Use arrows to specify direction on trajectories. Classify the stability of the equilibrium solution $\mathbf{x} = \mathbf{0}$.

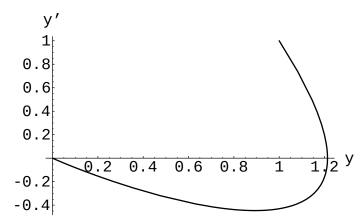
   (c) **(10 points)** Show how the system of differential equations $\mathbf{x}' = A\mathbf{x}$, for $\mathbf{x}^T(t) = [x_1(t), x_2(t), \ldots, x_n(t)]$, can be reduced to an eigenvalue problem for a constant $n \times n$ matrix A.

5. **(15 points)** Match each of the three differential equations with the appropriate phase-plane trajectory for the initial conditions $y(0) = y'(0) = 1$. Only your final answer will be graded on this problem (A,B,C,D,or E for each equation).

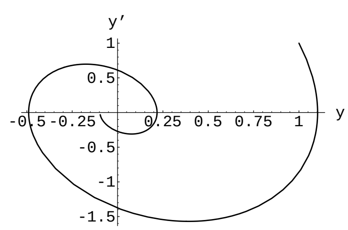
(1) $\dfrac{d^2y}{dt^2} + \dfrac{dy}{dt} + 4y = 0,$ (2) $\dfrac{d^2y}{dt^2} + 2\dfrac{dy}{dt} + y = 0,$ (3) $\dfrac{d^2y}{dt^2} - 1\dfrac{dy}{dt} + 2y = 0$



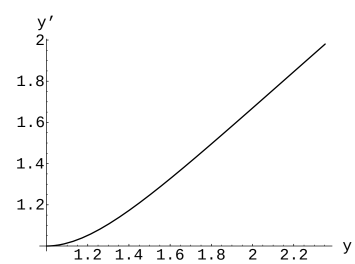
(A)



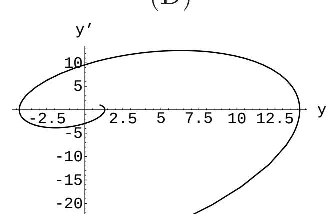
(B)



(C)



(D)



(E)