Math 24
Exam 1: February 21, 2007

ON THE FRONT OF YOUR BLUEBOOK WRITE (1) YOUR NAME, (2) A **FIVE**-PROBLEM GRADING GRID. **Show ALL of your work** in your bluebook, and **box in your final answers.** A correct answer, but without the relevant work, will receive no credit. You are allowed a one-page crib sheet. Start each problem on **the top of a new page.** Each problem is worth 20 points, for a total of 100 points. You can solve the problems in any order you like.

1. Classify the following equations as best you can:

$$\text{(a) } \frac{dy}{dt} = \frac{1+t^2}{y} \ ,$$
$$\text{(b) } \frac{dy}{dt} = \frac{y}{1+t^2} \ .$$

2. Answer the following TRUE/FALSE questions (write the word TRUE or FALSE; you do not need to show your work for this problem):

   (a) $y(x) = \cos x$ is a solution of the equation

$$\frac{d^2y}{dt^2} + (\sin^2 x)y + y^3 = 0 \ .$$

   (b) Picard's theorem guarantees the local existence and uniqueness of a solution of

$$\frac{dy}{dx} = \frac{y}{x} \ , \quad y(0) = 1 \ .$$

   (c) $\mathcal{L}x = 2x + 1$ is a linear operator.

3. Consider the IVP $\dfrac{dy}{dt} = (a-y)^2, \quad y(0) = y_0$, where a is a positive constant, and answer the following questions:

   (a) Sketch the phase lines and directions fields.
   (b) Classify the stability type of the equilibrium point(s).
   (c) Find the solution of the IVP.
   (d) How does the solution behave when $y_0 = a + 0.1$?

4. Find the general solution of $\dfrac{dy}{dx} - y = 3e^x$.

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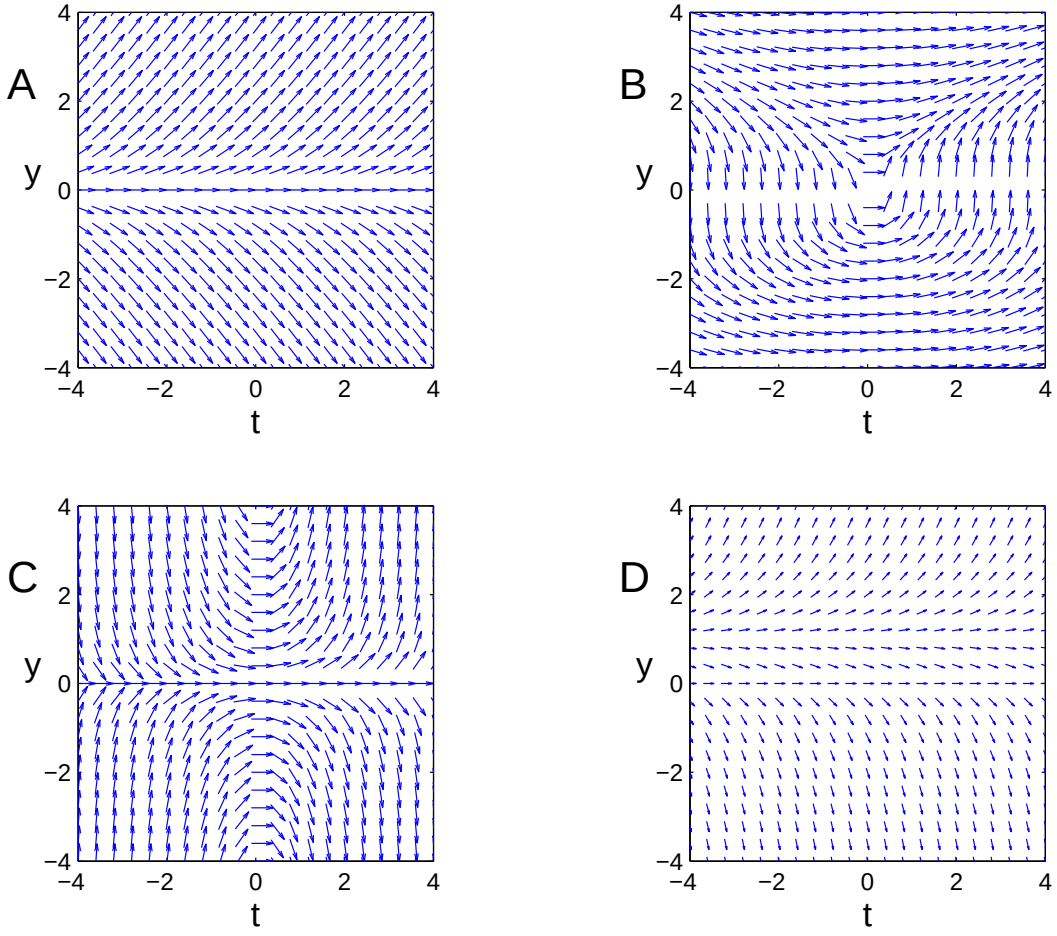
5. Match the following equations (1)–(4) with their corresponding direction fields A–D.

(1) $\dfrac{dy}{dt} = ty$, (2) $\dfrac{dy}{dt} = \dfrac{t}{y^2}$, (3) $\dfrac{dy}{dt} = y - y^{\frac{1}{3}}$, (4) $\dfrac{dy}{dt} = \tan^{-1} y$.



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