Math 30: Final Exam

Fall Semester 2006

Instructions. Read each problem carefully and follow all of its instructions. For each of the problems below, write a clear and concise solution in your blue book. For any short answer questions, write clearly your answer and any additional explanation that is needed.

1. (5 points) Find $\int x \sin(2x) \mathrm{d}x$.

2. (5 points) Find $\int_{-2}^{2} f(x) \mathrm{d}x$, where $f(x) = \begin{cases} 1 & x \leq 1, \\ x & x > 1. \end{cases}$

3. (5 points) Find $\int \frac{1}{\sqrt{1-4x^2}} \mathrm{d}x$.

4. (5 points) Find the arc length of the curve given parametrically as $x = 2\sin(t)$ and $y = 2\cos(t)$ for $0 \leq t \leq 2\pi$.

5. (5 points) Consider the region bounded by the curves $y = x$ and $y = x^2$. Find the volume of the solid generated by rotating that region about the y-axis.

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6. (5 points) Is the improper integral

$$\int_0^\infty \frac{\sin^2(x)}{(1+x)^2}\mathrm{d}x$$

convergent or divergent? Give reasons for your answer.

7. (5 points) Using the method of partial fractions, show that

$$\int \frac{2x+1}{2x^2+4x}\mathrm{d}x = \frac{1}{4}\ln|x| + \frac{3}{4}\ln|x+2| + C.$$

8. (5 points) Show that the function $y(x) = x^2 + \dfrac{1}{x^2}$ is the solution of the initial value problem

$$x\frac{dy}{dx} + 2y = 4x^2, \quad y(1) = 2.$$

9. Consider the system of equations

$$\frac{dx}{dt} = y - x^2$$
$$\frac{dy}{dt} = x - 1.$$

   (a) (5 points) Find the null clines and sketch them on the x-y plane. Then identify and label the equilibrium points on that sketch.

   (b) (5 points) Determine the direction of the flow on the null clines and add them to the sketch from part (a).