Midterm 2: Math 30, 10/21/07

Evaluate the following indefinite integrals. Answers must be in terms of x. Don't forget the constant!

4 pts 1) $\int xe^x dx$

$u = x \qquad dv = e^x dx$

$du = dx \qquad v = e^x$

$xe^x - \int e^x dx = \boxed{xe^x - e^x + C}$

4 pts 2) $\int \cos^4(x)\sin^3(x)dx$

$= \int \cos^4(x)\sin^2 x \underbrace{\sin x\, dx}_{du} \qquad u = \cos x$

$du = -\sin x\, dx$

$= -\int u^4(1-u^2)du = \frac{-u^5}{5} + \frac{u^7}{7} + C = \boxed{\frac{\cos^7 x}{7} - \frac{\cos^5 x}{5} + C}$

4 pts 3) $\int x\sqrt{x^2+4}dx$

$u = x^2+4 \qquad du = 2x\,dx$

$\frac{1}{2}\int \sqrt{u}\,du = \frac{1}{2}\cdot\frac{2}{3}u^{3/2} = \boxed{\frac{1}{3}(x^2+4)^{3/2} + C}$

4 pts 4) $\int \frac{1}{\sqrt{9-x^2}}dx$

$x = 3\sin\theta$

$dx = 3\cos\theta\, d\theta$

$\int \frac{3\cos\theta}{\sqrt{9-9\sin^2\theta}}d\theta = \int \frac{3\cos\theta}{3\cos\theta}d\theta = \theta + C$

$= \boxed{\sin^{-1}\left(\frac{x}{3}\right) + C}$

4 pts 5) $\int \frac{x-1}{x^2+6x+8}\,dx$

$$\frac{x-1}{x^2+6x+8} = \frac{x-1}{(x+4)(x+2)} = \frac{A}{x+4} + \frac{B}{x+2}$$

$$x-1 = A(x+2) + B(x+4)$$

Let $x=-2$

$$-3 = 2B \qquad \boxed{B = -\frac{3}{2}}$$

$x=-4$

$$-5 = -2A \qquad \boxed{A = \frac{5}{2}}$$

$$\int \frac{5}{2(x+4)}\,dx - \int \frac{3}{2(x+2)}\,dx$$

$$= \boxed{\frac{5}{2}\ln|x+4| - \frac{3}{2}\ln|x+2| + C}$$