Midterm 3: Math 30, 11/19/07

5 pts 1) Find the length of the curve. Give the final number for full credit.

$$y = \left(4 - x^2\right)^{1/2}, 0 < x < 1$$

5 pts 2) Find the area of the surface obtained by rotating the curve about the x-axis.

$$x = \frac{1}{3}\left(y^2 + 2\right)^{3/2}, \ 1 \le y \le 3$$

5 pts 3) Find the solution of the differential equation $(x^2 + 1)\dfrac{dy}{dx} = xy$ that satisfies the initial condition $y(1) = 1$. Solve for y explicitly for full credit.

5 pts 4) A bacteria culture grows at a rate of $\dfrac{dP}{dt} = \lambda P$.

a) Find the solution for P(t) given the initial condition $P(0) = P_o$ (You can guess the solution but show that it satisfies the differential equation)

b) How long does it take for the initial population to double?

5 pts 5) A function $y(t)$ satisfies the differential equation. $\dfrac{dy}{dt} = y^4 - 12y^3 + 35y^2$

a) Find and plot equilibrium points
b) Determine whether equilibrium points are stable or unstable (or perhaps something else).

Extra Credit(5 pts)

Solve the differential equation.

$$y' = xe^{-\sin x} - y\cos x$$