Midterm 1, Math 30, Fall 2008, 10/6/08

1) Find the volume of a solid when the region bounded by $y = \sin(x)$ and y=0 is revolved around the y-axis (10 pts)

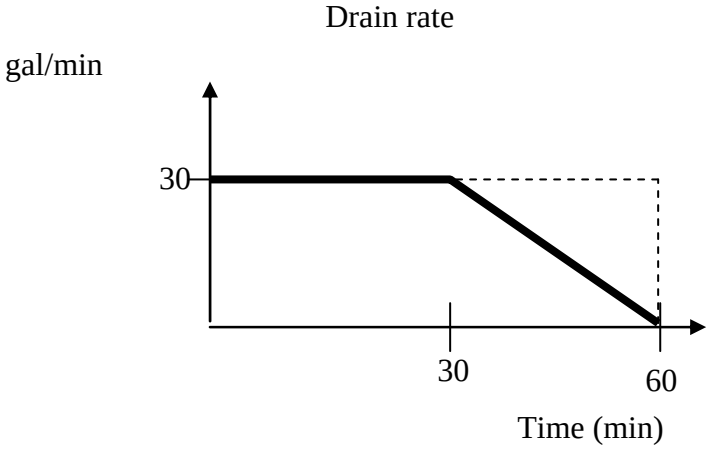
2) An enclosure for rare reptiles must maintain an average temperature of 30C. The temperature profile during the day is given by $T(t) = T_m \cos(\frac{\pi}{48} t)$ where t is in hours. Find T_m so that the average temperature during 24 hours is 30C. (10 pts)

Evaluate the following integrals: Answer must be in terms of x

3) $\int \frac{\sqrt{x^2 - 1}}{x} dx$ (10 pts)

4) $\int \frac{3x + 5}{x^2 + x - 6} dx$ (10 pts)

5) A tank with 2000 gal of water is being drained at the rate given in the diagram. How much water is left in the tank after 50 min ? (12 pts)



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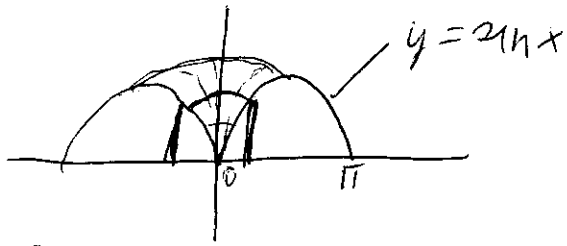
(1)



shell method $V = 2\pi \int_0^{\pi} 2 y \, dx$

$r = x$, $y = \sin x$

$$V = 2\pi \int_0^{\pi} x \sin x \, dx$$

$u = x$ $\quad dv = \sin x$

$du = dx$ $\quad v = -\cos x$

$$V = 2\pi \left(-x\cos x \Big|_0^{\pi} + \int_0^{\pi} \cos x \, dx \right)$$

$$= 2\pi \left(-x\cos x + \sin x \right)\Big|_0^{\pi} = 2\pi(\pi) = 2\pi^2$$

(2) $T_{Av} = \frac{1}{24} \int_0^{24} T_m \cos\left(\frac{\pi t}{48}\right) dt = 30$

$$= \frac{T_m}{24} \frac{48}{\pi} \sin\left(\frac{\pi t}{48}\right)\Big|_0^{24} = \frac{2T_m}{\pi} = 30$$

$T_m = 15\pi$

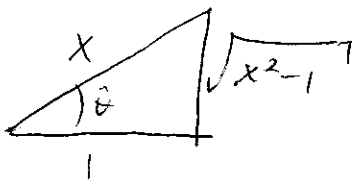
[redacted]

(3) $\int \frac{\sqrt{x^2-1}}{x}\,dx$ $\quad x = \sec\theta$

$dx = \sec\theta\tan\theta\,d\theta$

$= \int \frac{\tan\theta \sec\theta\tan\theta}{\sec\theta}\,d\theta = \int \tan^2\theta\,d\theta$

$= \int (\sec^2\theta - 1)\,d\theta = \tan\theta - \theta$



$= \boxed{\sqrt{x^2-1} - \operatorname{arcsec}\left(\frac{1}{x}\right) + C}$

(4) $\int \frac{3x+5}{x^2+x-6}\,dx$ $\quad \frac{3x+5}{x^2+x-6} = \frac{3x+5}{(x+3)(x-2)} = \frac{A}{x+3} + \frac{B}{x-2}$

Common denominator: $3x+5 = A(x-2) + B(x+3)$

$3x+5 = (A+B)x + (3B-2A)$

$A+B=3$ $\quad * -3 \Rightarrow -3A-3B=-9$

$3B-2A=5$ $\quad 3B-2A=5$

$-5A = -4$

$\boxed{A = \frac{4}{5}}$

$B = 3-A = 3-\frac{4}{5} = \boxed{\frac{11}{5}}$

$$\boxed{\int \frac{(3x+5)}{x^2+x-6}\,dx = \frac{4}{5}\ln|x+3| + \frac{11}{5}\ln|x-2| + C}$$

(5) Amount of water drained from tank

$$\int_0^{50} r(t)\,dt \qquad r(t) = \begin{cases} 30 & 0 < t < 30 \\ -(t-60) & t \geq 30 \end{cases}$$

$$\int_0^{50} r(t)\,dt = \int_0^{30} 30\,dt + \int_{30}^{50} 60 - t\,dt$$

$$= 30t\Big|_0^{30} + \left(60t - \frac{t^2}{2}\right)\Big|_{30}^{50}$$

$$= 900 + \left(3000 - \frac{2500}{2} - 1800 + \frac{900}{2}\right)$$

$$= 1300$$

Amount of water left $2000 - 1300 = 700$ gal