Final Exam, Math 30, May 16, 2008

Solve for y explicitly. Indefinite integrals must be in terms of x. Check out useful information in the Appendix.

1) Find the volume of the solid obtained by rotating the given region about the y-axis. **(10 pts)**
Region bounded by: $y = \ln(x), 1 \le x \le 3$

Evaluate the following indefinite integrals **(10 pts each)** solutions must be in terms of x.

2) $\int \frac{\sin(\ln x)}{x} dx$

3) $\int \frac{1}{x^2\sqrt{x^2+5}} dx$

4) Calculate the constant c for the probability density function p(x) **(10 pts)**

$$p(x) = c(5x - x^2)\ 0 \le x \le 5$$
$$p(x) = 0\ x < 0\ and\ x > 5$$

5) Find the solution to the differential equation **(15 pts)**

$$\frac{dy}{dx} = \frac{y}{x^2+5x+6}, \quad y(0) = 2$$

6) Air freshener's (AF) evaporation rate is given by $\frac{dm}{dt} = -\lambda m, \quad \lambda = .02/day$ and m is the mass. If after 10 days 10.0 mg of AF is left, what was the starting mass of AF? **(10 pts)**

7) A climate model for carbon dioxide concentration in atmosphere (C) in ppm is represented by the following equation.

$$\frac{dC}{dt} = k(C-450)^2(400-C)(350-C)$$

a) Find and identify by type all equilibrium points **(10 pts)**

b) Plot the solution if the starting concentration is 410 ppm. **(4 pts)**
c) Is there anything troubling about 450 ppm concentration? **(3 pts)**
d) By what minimum amount would you reduce the starting C to have a stable concentration over time? **(3 pts)**

8) Consider a population P=P(t) where m and λ are constants representing immigration rate and growth rate. m is always positive and λ can be either positive or negative.

a) Solve the differential equation **(10 pts)**

$$\frac{dP}{dt} = \lambda P + m, \quad P(0) = P_o$$

Find the conditions on λ in terms of m and P_o that will lead population to:

b) Grow **(2 pts)**
c) Decline **(2 pts)**
d) Stay constant **(2 pts)**

9) Write down but don't solve the differential equations (DEs) for the following problems. Pay close attention to the wording. **Proportionality constant k is always positive. Make sure that you have correct signs. Analyze what is changing and what is driving the change.**

a) During a disease outbreak in a town the number healthy people decreases proportionally to the product of healthy people and sick people. Write the DE for the rate at which the number of healthy people decreases in terms of H (number of healthy people), T(Total number of people) and k. **(7 pts)**

b) Cooling rate of a hot cup of coffee is proportional to difference between coffee temperature (T) and ambient air temperature (T_a). Write the DE for the rate at which coffee is cooling in terms of T, k, and T_a. **(7 pts)**

10) For the following predator pray system determine which of the variables, x or y, represent the prey population and which represent the predator population (Explain) **(5 pts).** Find equilibrium solutions for predator and prey. **(10 pts)**

$$\frac{dx}{dt} = -.02x + .00002xy$$

$$\frac{dy}{dt} = 0.1y - .001xy$$

Appendix:

Identities:

$$\sec^2(x) = \tan^2(x) + 1$$

$$\cos^2(x) + \sin^2(x) = 1$$

$$\sin^2(x) = \frac{1}{2}\left(1 - \cos(2x)\right)$$

$$\frac{d\tan(\theta)}{d\theta} = \sec^2(\theta)$$

$$\frac{d\sec(\theta)}{d\theta} = \tan(\theta)\sec(\theta)$$