Duration: 50 minutes
Instructions: A cheat sheet on one side of a 8.5x11" page is allowed and must be turned in with the exam. A calculator is allowed as well. Partial credit will be awarded for correct work, unless otherwise specified. The total number of points is 100.

1. (25 points: 5 each) For each of the following problems, identify the type of random variable as well as the values of all parameters. If you forget the name of the distribution, you may write down the appropriate PDF or PMF. You do not have to write out reasoning for this problem.

   Hint: Possible answers are the following: Bernoulli(p), Binomial(n, p), Poisson(λ), Hypergeometric(N, M, n), Uniform($[a, b]$), Normal(μ, σ), Exponential(λ), Chi-Square(n), t–distribution(n), or F–distribution(n, m).

   (a) $X = X_1 + X_2$ where both X_1 and X_2 are independent Poisson random variables with mean 3.

   (b) H is the number of heads in 40 tosses of a fair coin.

   (c) A club contains 50 members; 20 are men and 30 are women. A committee of 10 members is chosen at random. Let W denote the number of women on the committee.

   The next two parts refer to a sample $X_1, X_2, \ldots, X_n$ from a normal population with mean 4 and variance 100. $\bar{X} = (X_1 + X_2 + \cdots + X_n)/n$ is the sample mean and $S^2 = (\sum_{i=1}^{n}(X_i - \bar{X})^2)/(n-1)$ is the sample variance.

   (d) The sample mean $\bar{X}$.

   (e) $\sqrt{n}\dfrac{(\bar{X} - 4)}{S}$.

2. (20 points) Suppose that at a certain garage, the time, in minutes, required to tune up a passenger car is uniformly distributed over the interval $[20, 45]$. One day, this garage received tune-up requests on 6 such passenger cars. What is the probability that exactly 2 of the 6 cars take less than 30 minutes to tune up?

3. (15 points) The yearly income, X, in thousands of dollars, for families in a certain city, is normally distributed with mean $\mu = 53$ and variance $\sigma^2 = 25$. The city imposes a city income tax of 1% for the amount above 20K. That is, the city income tax for each family is given by the formula $T = 0.01(X - 20)$ thousands of dollars. Find the expectation and variance of T.

4. (20 points) Find the moment generating function of the following random variable and use your result to find the expectation and variance. The probability density function of the random variable X is given as

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0 \\ 0, & \text{otherwise,} \end{cases}$$

   for some constant λ. (X is called an exponential random variable with parameter λ.)

   !!! CONTINUE ON THE BACK !!!

5. (20 points) A surveyor is measuring the height of a cliff known to be about 1000 feet. He assumes his instrument is properly calibrated and that his measurement errors are independent, with mean $\mu = 0$ and variance $\sigma^2 = 10$. He plans to take n measurements and form the average. Estimate, using

   (a) the central limit theorem

   how large n should be if he wants to be 95% sure that his average falls within 1 foot of the true value.

TABLE A1 *Standard Normal Distribution Function:* $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-y^2/2}\, dy$

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998