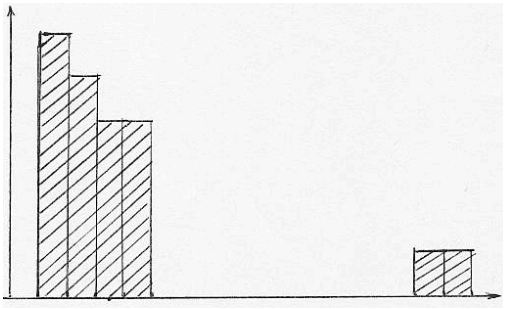
Duration: 50 minutes
Instructions: Answer all questions, without the use of notes, books or calculators. Partial credit will be awarded for correct work, unless otherwise specified. The total number of points is 80.

1. (25 points: 5 each) Only final answers to this problem are going to be graded.

   (a) The following is the histogram of a data set.

   Compare the mean, median, and mode—which one is the largest, which one is the smallest?

   (b) True or False: If we have a data set containing 100 values with sample mean 11 and sample standard deviation 5, then according to Chebyshev's Inequality at least 75 of those 100 values lie in the interval $[1, 21]$.

   (c) True or False: $P(E \cup F) = P(E) + P(F)$ for any events E and F.

   (d) Given that random variable X has expectation $E[X] = -2$ and variance $\mathrm{Var}(X) = 3$ and that $Y = 4X + 3$, calculate $E[Y]$ and $\mathrm{Var}(Y)$.

   (e) Sketch 3 different scatter diagrams, each with at least 5 data points, with the following three correlation coefficients respectively. Your three diagrams only need to look qualitatively correct and correct relative to each other. (That is, you do not need to calculate exact numbers.)

      i. $r = -1$.
      ii. $r = 0.9$.
      iii. $r = 0.1$.

2. (15 points total) The route used by a certain motorist in commuting to work has two intersections with traffic signals. The probability that he must stop at the first signal is 0.4. The probability that he must stop at the second signal is 0.5. The probability that he must stop at at least one of the signal is 0.6.

   (a) (7 points) What is the probability that he must stop at both signals?

   (b) (8 points) Are "stopping at the first signal" and "stopping at the second signal" independent events and why?

CONTINUE ON THE BACK!

3. (25 points total) The lifetime in hours of a certain kind of radio tube is a random variable having a probability density function

$$f(x) = \begin{cases} 0, & x \leq 100, \\ \dfrac{k}{x^2}, & x > 100. \end{cases}$$

   (a) (6 points) Find the value of k.

   (b) (6 points) Write down an expression to calculate the expected lifetime of this kind of tubes. Do not evaluate!

   (c) (6 points) What is the probability that a randomly selected tube will have to be replaced within the first 150 hours of operation? You need to calculate the number for this one.

   (d) (7 points) Assume that each tube needs to be replaced independently. What is the probability that exactly 2 of 5 such tubes in a radio set will have to be replaced within the first 150 hours of operation?

4. (15 points total) There are 30 red marbles and 19 blue marbles in a jar. You randomly pick 3. Let R be the number of red marbles among the 3 which you picked. Write down expressions to answer the following questions, but do not evaluate.

   (a) (8 points) Find the probability mass function of R.

   (b) (7 points) What is the expected value of R?