Duration: 50 minutes
Instructions: Answer all questions, without the use of notes or books. Calculators may be used to calculate numbers only. Partial credit will be awarded for correct work, unless otherwise speci-fied. The total number of points is 70.

1. (15 points total)

    (a) (8 points) Jones figures that the total number of thousands of miles that a used auto can be driven before it would need to be junked is an exponential random variable with parameter $1/20$. Smith has a used car that he claims has been driven only 10,000 miles. If Jones purchases the car, what is the probability that she would get at least 20,000 additional miles out of it?

    (b) (7 points) Repeat under the assumption that the lifetime mileage of the car is not ex-ponentially distributed but rather is (in thousands of miles) uniformly distributed over $(0, 40)$.

2. (15 points) X is a Bernoulli random variable with parameter p. That is, X may take on two values only, 0 and 1, and $P\{X = 1\} = p$ and $P\{X = 0\} = 1 - p$. Find the moment generating function of X and use it to find $E[X]$ and $\text{Var}(X)$.

3. (25 points total) If you buy a lottery ticket in 50 lotteries, in each of which your chance of winning a prize is $1/100$.

    (a) (8 points) What is the exact probability that you will win a prize at least twice?

    (b) (7 points) Use Poisson distribution to approximate the probability in part (a).

    (c) (10 points) Use the Central Limit Theorem to estimate the probability in part (a). (You do not have to use continuity correction.)

4. (15 points: 3 each)

    (a) Suppose that X and Y are random variables with $E[X] = 5$ and $E[Y] = -2$. Find $E[3X + 4Y - 7]$.

    (b) Suppose that $\text{Var}(X) = 5$ and $\text{Var}(Y) = 9$. Is there enough information to find $\text{Var}(3X + 4Y - 7)$? If yes, what is your answer? If not, what else would you need to know?

    (c) Describe one situation where you see a T–distribution.

    (d) Give an example of two independent random variables which have the same kind of distributions and the sum of which also has the same kind of distribution.

    (e) Besides calculating the expectation and variance, what other kind(s) of problems can you solve using moment generating functions?

TABLE A1 *Standard Normal Distribution Function:* $\Phi(x) = \dfrac{1}{\sqrt{2\pi}} \displaystyle\int_{-\infty}^{x} e^{-y^2/2}\, dy$

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998