Preliminary Exam: Calculus

University of California, Merced, January 2026

EXAM INSTRUCTIONS AND POLICIES

- You have 4 hours to answer all 8 questions on this exam.
- In order to receive full credit, you must show your work and carefully justify your answers, unless otherwise specified. Correct answers without work will receive little or no credit.
- You may use a one-sided, one page sheet of your own hand-written notes for this exam. No other notes, books, or calculators are permitted.

---

1. Evaluate the following limits:

   (a) $\lim_{x\to 0^+} x \ln x$

   (b) $\lim_{x\to 0^+} x^{1/x}$

2. Evaluate the following integrals:

   (a) $\int_0^{\pi} x \cos x \, dx$

   (b) $\int \frac{x^5}{(1-x^3)^{3/2}} \, dx$

3. For each of the following series, determine if they converge or diverge. **Be sure to provide complete explanations for your conclusions. In particular, you must state the name of every convergence test/strategy you use, and why the conditions of those tests are met.**

   (a) $\sum_{n=1}^{\infty} \frac{e^n}{n!}$

   (b) $\sum_{n=1}^{\infty} \frac{\sin(1/n)}{n^2}$

4. A soda company wants its aluminum soft drink cans (cylindrical in shape) to have a volume of 128π cubic centimeters. The company's factory can only manufacture cans that are at least 2 cm tall and have a radius of at least 1 cm. In order to conserve resources, the company wants to minimize the amount of aluminum needed for a single can. What dimensions should they make their cans?

5. For the following iterated integral:

$$\int_0^{\sqrt{2}} \int_0^{2-x^2} xe^{x^2} \, dy \, dx$$

   (a) Sketch a picture of the region of integration.

   (b) Change the order of integration.

   (c) Evaluate the integral (the original or the one with the changed order, whichever is easier).

6. Use a triple integral to find the volume of the solid region that is outside the cone $z = \sqrt{x^2 + y^2}$ and inside the paraboloid $z = x^2 + y^2$.

7. Consider $\vec{F}(x, y, z) = \langle ze^{xz} + \cos x, z^2, xe^{xz} + 2z + 2yz \rangle$.

   (a) Is $\vec{F}$ a conservative vector field? Explain.

   (b) Evaluate $\displaystyle\int_C \vec{F} \cdot d\vec{r}$, where C is a linear path from $(0, 0, 0)$ to $\left(\frac{\pi}{2}, 1, 1\right)$.

8. Let $\vec{F} = \langle \sin(yz),\ 4e^{x^2 z},\ 3z^2 \rangle$, and let S be the surface of the hemisphere $z = \sqrt{4 - x^2 - y^2}$ together with the disk $x^2 + y^2 \leq 4$ in the xy-plane.

   Use divergence theorem to evaluate $\displaystyle\iint_{\mathcal{S}} \vec{F} \cdot d\vec{S}$.